Signal processing approaches to obtain complex resistivity and phase at multiple frequencies for the electrical exploration method

LIU WEI-QIANG, WANG JUN-LU and LIN PIN-RONG

Institute of Geophysical and Geochemical Exploration, Chinese Academy of Geological Sciences, Hebei, China

(Received: April 6, 2016; accepted: April 11, 2017)

ABSTRACT The electrical method is an effective geophysical exploration method. Complex resistivity and phase at multiple frequencies can be obtained simultaneously by transmitting pseudo random signal current (2^n-sequence or m-sequence). However, with the development of industry, electromagnetic (EM) interference has been a challenging problem to the transmitted signal, including noise interference and EM coupling. This article describes the application of several simple and effective signal processing approaches to improve the data quality for the electrical method. For the problem of noise interference, a robust statistical method was studied. In addition, coherence analysis between current data and potential data was applied to extract data with high signal-to-noise ratio. For EM coupling interference, a relative phase spectrum was used to remove coupling interference in the complex resistivity phase and an inverse differential operation was adopted to correct amplitude spectrum. We demonstrated the effectiveness of our processing approach by showing examples of data acquired in a noisy survey area for electrical prospecting. Compared to common methods, these processing methods can suppress EM interference effectively and do not lead to signal distortion. These approaches can be considered for data processing in electrical prospecting.

Key words: electrical exploration, complex resistivity, signal processing, noise interference, electromagnetic coupling.

1. Introduction

Earth science is composed of many disciplines. Geophysics is one of the major ones. The electric method, seismic method, gravity method, and magnetic method are common methods in geophysical exploration (Oskooi et al., 2013; Aretouyap et al., 2016). This paper describes several signal processing methods for electrical exploration.

Recently, there has been a major development in the electric method, in that pseudo random signal has been applied in electrical exploration. Amplitude and phase of complex resistivity at multiple frequencies can be obtained at the same time with only one power supply by transmitting this current.

Electrical methods are widely used in environmental and engineering exploration (Keller, 1975; Herman, 2001; Matias, 2003). Because targets are usually buried underground and cannot
be directly observed, it is necessary to acquire the potential difference signal on the surface of the Earth after injecting current. Apparent complex resistivity and phase can be calculated using potential difference data and current data. The underground target information can be collected through signal processing, forward modelling, and inverse interpretation. However, the original potential signal is contaminated by electromagnetic (EM) interference from a number of different sources, including industrial current interference, major power lines, impulsive stray current, high-amplitude spheric pulses, and so on (Robinson and Treitel, 1980).

Signal processing is significant for electric prospecting (Szarka, 1988). Digital filter and mean stacking are commonly used for noise suppression in various electrical instruments (Cruciani and Monna, 1992; Buselli and Cameron, 1996; Olsson et al., 2015). Digital filters based on Discrete Fourier Transform (DFT) are widely used to suppress noise, including high-pass, low-pass, band-pass, and notch filter (Smith, 2003). However, interference in electrical potential signal is mixed with many different types of noises, and there is a large overlap between noise spectrum and signal spectrum, which makes the digital filter useless. Mean stacking can suppress Gaussian noise and occasional spike impulse noise by extending the observation time, but cannot suppress spike impulse noise appearing repeatedly. Moreover, mean stacking cannot evaluate the quality of each periodic data set and select the data with high signal-to-noise ratio (SNR). The median filter can suppress the spike impulse noise effectively but cannot suppress the Gaussian random noise (Liu et al., 2016). Other signal processing methods also may cause signal distortion when EM interference is serious. So signal processing in a strong interference environment is still a challenging subject for electrical exploration (Liu et al., 2016; Olsson et al., 2016). Simple and effective anti-interference methods should be studied.

In order to suppress noise interference and extract the real signal, robust statistics similar to the maximum likelihood estimate are studied and applied to the process signal for electrical prospecting. Compared to ordinary mean stacking, robust stacking is effective at reducing outliers caused by spike impulse noise and improving data quality (Egbert and Booker, 1986; Buselli and Cameron, 1996; Liu et al., 2016). This processing does not lead to signal distortion.

In addition, we present a new technique for extracting high-quality data from noisy long-time acquisition by analysing the coherence of potential difference and transmitting current. It is also an effective method for extracting data with a high SNR from the original data (Lamarque, 1999).

EM coupling between the current supply line and the Earth is also a major impediment to the interpretation of complex resistivity (Routh and Oldenburg, 2001). Based on previous research findings, two methods are studied to correct the EM coupling for complex resistivity. The relative phase spectrum is used to remove coupling interference in the complex resistivity phase spectrum. The inverse differential operation is adopted to correct amplitude spectrum. We demonstrate the effectiveness of our processing approach by showing examples of data acquired in a very noisy survey area for electrical prospecting.

2. Electrical method based on pseudo random signal

There has been a major development in the electrical method recently. The electrical method based on measuring complex resistivity at multiple frequencies is also called alternating-current
electrical method or frequency-domain induced polarization method (Wait, 1959; Herman, 2001). The electrical method based on the pseudo random signal has been proposed and studied by many geophysical experts (Duncan et al., 1980; He et al., 2009; Ilyichev and Bobrovsky, 2015). Complex resistivity and phase at multiple frequencies can be obtained simultaneously by transmitting pseudo random signal current. The m-sequence is a kind of binary sequence generated by using maximal linear feedback shift registers. Practical applications of the m-sequence are in digital communication systems that employ direct-sequence spread spectrum and frequency-hopping spread spectrum transmission systems (Golomb, 1994). We have acquired electric potential difference data through transmitting 5-order m-sequence current. The 5-order m-sequence current and amplitude spectrum at 0 Hz - 1 Hz are shown in Fig. 1.

![Fig. 1 - 5-order m-sequence current: a) waveform corresponds to one period, b) amplitude spectrum at 0 - 1 Hz.](image)

In Fig. 1, compared to other signals, the power of the m-sequence signal is more uniform (flat) at frequencies 1/16 Hz, 2/16 Hz, 3/16 Hz, 4/16 Hz, …., 1 Hz with a linear interval. The $2^n$-sequence is also a kind of pseudo random signal, which was invented by He (2009). The amplitude of the $2^n$-sequence signal at $2^0$ - $2^n$ Hz is substantially equable. An example of this signal, $2^7$-sequence, and the amplitude spectrum are shown in Fig. 2.

In Fig 2, compared to the m sequence, the power of the $2^7$-sequence signal is more uniform at frequencies $2^0$ Hz, $2^1$ Hz, $2^2$ Hz, $2^3$ Hz, …., $2^7$ Hz with a logarithmic interval. Both m-sequence and $2^n$-sequence are broadband signals. By transmitting pseudo random signal current in electrical prospecting, complex resistivity with a similar SNR at multiple frequencies can be calculated at the same time with only one power supply, which overcomes the shortcomings of repeated power supplies for different frequencies. The main advantage of transmitting pseudo random signal current is the improvement of working efficiency, namely, measuring time decreases massively, compared to excitation at a single frequency each time. In addition, when a single-
When a pseudo random current is supplied, complex resistivity at multiple frequencies is measured at the same time, so they can be processed simultaneously.

3. Robust statistics for reducing noise

Robust stack is a simple and effective method which can be used in electrical data processing to suppress outliers caused by noise (Huber, 1981). For observations in periods: $Y_i, i=1,\ldots, N$, the robust M-estimate $\theta$ can be calculated by solving the equation:

$$\sum_{i=1}^{N} \psi \left( \frac{Y_n - \theta}{\sigma} \right) = 0 \quad (1)$$

where $\psi$ is the influence function, which represents a class of functions that determine the influence of original observations in calculating the estimator (Holland and Welsch, 1977; Street et al., 1988). $\sigma$ is the scale parameter representing residual distribution range. Eq. 1 can be solved through an iterative algorithm (Huber, 1981).

In order to compare the robust stack and mean stack, we transmitted m-sequence current and acquired full-waveform potential data at a survey point beside a mine in Gansu province, China. The equipment is multi-frequency, multi-function for the electrical system (Chen et al., 2007).
The transmitter generates 5-order m-sequence as source current with the sampling rate of 64 Hz and 1024 data per period. The value of the current is 8 A, and the base frequency is 1/16 Hz. For this survey station, current electrodes A and B are located at 0.2200 m and 2200 m, the potential electrodes M and N are located at 760 and 780 m. We obtained original data of a number of sufficient periods (160 periods) by measuring repeatedly for a long time (more than 40 minutes). The robust M-estimate can be used to stack potential difference data and current data of all the periods. Then we can calculate complex resistivity at 16 frequencies by using the formula (Zonge and Wynn, 1975):

\[
\bar{\rho} = K \frac{\bar{U}}{\bar{I}}
\]

where \(\bar{U}\) was the frequency spectrum of potential difference data, \(\bar{I}\) was the frequency spectrum of current data, \(K\) was the setting coefficient determined by the spacing of electrodes A, B, M, N, \(\bar{\rho}\) was complex resistivity.

We observed 160 periods of electrical data. In order to study the convergence of calculated results with stack times, we calculated the complex resistivity and error of stacking 10 times to 160 times by mean stacking and robust stacking. Fig. 3 shows the convergence of the complex resistivity at one frequency with stack times using the common mean stack and robust stack.

Fig. 3 - Convergence of complex resistivity with stack times for different stacking methods: a1) amplitude at 5/16 Hz; a2) phase at 5/16 Hz; b1) amplitude at 12/16 Hz; b2) phase at 12/16 Hz.
In Fig. 3, the convergence of ordinary mean stacking was poor. Results of different stack times were also different. For the robust methods, the outliers in the original data were down-weighted, and the calculated result was more accurate than when stacking fewer times. Robust processing can improve the quality of Session Initiation Protocol (SIP) data and save measuring time in the field.

4. Coherence analysis for exact, high-quality data

Robust stacking is efficient to reduce outliers caused by noise interference. However, when the percentage of outliers in the overall acquired potential data exceeds breakdown (50%), this method is not effective. Another helpful method for electrical data processing is proposed in this paper, namely, coherence analysis. Coherence between current data and potential data was investigated to analyse the data quality. For multiple periods of electrical data, we calculated coherence of current data \( x_i \) and potential difference data \( y_i \) in each period using the following formula (Gayen, 1951):

\[
C = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}
\] (3)

The higher the coherence of current data and potential difference was, the higher the SNR of potential difference data was, and vice versa.

Fig. 4 - a1) current signal in the period showing coherence of 0.59; a2) potential signal in the period showing coherence of 0.59; b1) current signal in the period showing coherence of 0.01; b2) potential signal in the period showing coherence of 0.01.
In order to verify the effect of coherence analysis, we also used the current and potential data of 160 periods acquired at the survey point in Gansu province to calculate the coherence coefficient of current and potential in each period. Fig. 4 shows two signal segments with different coherence. From Fig. 4, for multiple periods, with potential difference data observed at the same survey point when the signal was in good condition, the coherence of current data and potential data was high, such as in Figs. 4a1 and 4a2. On the contrary, when the potential signal was contaminated by strong electromagnetic interference and background Gaussian noise, the coherence of current data and potential difference data was low, such as in Figs. 4b1 and 4b2. By using coherence analysis to extract data with high SNR and remove data with low quality, we can get more accurate complex resistivity spectra after subsequent processing.

5. Correcting methods to remove EM coupling interference

For electrical exploration, EM coupling between current supply line and the Earth is a kind of interference (Hallof, 1974). International experts have produced a lot of research findings related to decoupling, including EM coupling forward computing based on uniform terrestrial condition and layered medium, correction technique of multiple frequencies, fitting EM coupling using the Cole-Cole model, direct decoupling scheme by chop wave, and so on (Pelton et al., 1978; Brown, 1985; Katsarakis et al., 2004). In this paper, relative phase spectrum was used for decoupling in the complex resistivity phase spectrum, based on previous research findings. Inverse differential operation was proposed to correct amplitude spectrum at low frequencies. Details of these two methods follow.

These two decoupling methods were based on two assumptions (Millett, 1967; Sunde, 1968; Pelton et al., 1978): 1) at low frequencies (0.01 Hz - 10 Hz), change of complex resistivity phase spectrum is small, but EM coupling phase increases with frequency proportionally; 2) at low frequencies (0.01 Hz - 10 Hz), change of complex resistivity amplitude spectrum is similar to an oblique line, but EM coupling amplitude is N power of frequency. Both assumption (1) and (2) hold for data after stacking.

Relative phase spectrum can be used for decoupling in the complex resistivity phase spectrum (Chen et al., 2009). The formula is:

\[
\Delta \varphi = \frac{k \varphi_d - \varphi_g}{k - 1}
\]

where \( f_g \) and \( f_d \) were two frequencies; \( \varphi_g \) was the phase at higher frequency \( f_g \); \( \varphi_d \) was the phase at lower frequency \( f_d \).

The resistivity amplitude spectrum was approximately linear, and EM coupling amplitude was approximately nonlinear at low frequencies. The second derivative of the linear function was 0 and the nonlinear function was not 0. Calculating the second derivative of the complex resistivity amplitude spectrum through the forward difference and applying an inverse differential operation, we can get the nonlinear component. The linear component can be obtained by subtracting the nonlinearity, which represented the pure complex resistivity amplitude spectrum.

We use other practical data to illustrate the effectiveness of these two EM coupling correction methods. These practical current and potential data were acquired in a 2D survey line to detect
water-bearing fracture zones. The equipment was a multifunctional EM prospecting system, DEM, which was developed by the Institute of Geophysical and Geochemical exploration, Chinese Academy of Geological Sciences (Li et al., 2013). In this survey line, the intermediate gradient array protocol was centred on the top of a geological target. The current electrode distance AB is 2500 m and the potential electrode distance MN is 50 m. The survey line is 1250 m, consisting of 25 survey points. The transmitting current was combined 2\(^7\)-sequence. Complex resistivity at 2\(^7\)-2\(^7\) Hz was calculated using current and potential data after coherence analysis and robust stacking.

Using the relative phase spectrum and inverse differential operation methods to correct complex resistivity and phase at low frequencies of one survey point for phase at each frequency, the relative phase is calculated by Eq. 4. \(\varphi_d\) is the phase at this frequency and \(\varphi_g\) is the phase at the next frequency, \(k = 2\), because the ratio of two adjacent frequencies is 2. For the amplitude correction, we calculated the second derivative of the complex resistivity amplitude spectrum through forward difference, namely,

\[
\frac{dR_i}{d} = \text{diff}(R_i) = \frac{R_{i+1} - R_{i-1}}{\log(f_{i+1}) - \log(f_{i-1})}
\]

\[
sR_i = \text{diff}(dR) = \frac{dR_{i+1} - dR_{i-1}}{\log(f_{i+1}) - \log(f_{i-1})} \quad i = -7, -6, -5, \ldots, 7
\]

where \(R_i\) is the complex resistivity at 2\(^i\) Hz, and the difference is equal to the adjacent value when the \(i\) is -7 or 7. Because the second derivative of the linear function was 0 and the nonlinear function was not 0, the linear component (pure complex resistivity amplitude) disappeared in the Eq. 5. Then, by inverse differential operation, we can get the nonlinear component (the EM coupling in amplitude spectrum). Finally, the linear component (the pure complex resistivity amplitude spectrum) was obtained by subtracting the nonlinearity from the original amplitude spectrum. The original phase spectrum and corrected phase spectrum were shown in Fig. 5a. The original amplitude spectrum and corrected amplitude spectrum were shown in Fig. 5b.

EM coupling is strong at high frequencies and weak at low frequencies (Pelton et al., 1978). In Fig. 5, we can see that the correction is effective at 0.01 Hz - 10 Hz. Before correction, phase became the positive value and amplitude was a nonlinear function, which was caused by EM coupling. After correction, the relative phase spectrum became the negative value and the corrected amplitude decreased with frequencies similar to a linear function. These were pure complex resistivity spectra. However, when the frequency was larger than 10 Hz, the correction was not effective, because the assumptions were not satisfied.

In order to demonstrate the effectiveness of the correcting approach, we processed electrical data in a survey line. Complex resistivity amplitude, phase, and polarizability were calculated and corrected. The polarizability was calculated using resistivity amplitude at different frequencies by the formula (Wait, 1959):

\[
F_s = \frac{\rho_d - \rho_g}{\rho_g} \times 100\%
\]

where \(\rho_d\) and \(\rho_g\) are complex resistivity amplitude at one frequency and the next frequency, respectively.
In Fig. 6, a1, a2, a3 at the top, middle, and bottom represent the original phase, amplitude, and polarizability, respectively, at 0.01 Hz. Because the frequency is low, the EM coupling is weak (Pelton et al., 1978), and the original spectrum can be seen as the pure phase, amplitude, and polarizability.

In Fig. 6, b1, b2, b3 at the top, middle, and bottom represent the original phase, amplitude, and polarizability, respectively, at 1 Hz. Because the frequency is high, the EM coupling is strong (Pelton et al., 1978). In the same figure, c1, c2, c3 at the top, middle, and bottom represent the corrected phase, amplitude, and polarizability, respectively, at 1 Hz. Because the EM coupling is removed, the profiles are similar to the profile at 0.01 Hz, and these profiles can be seen as the pure complex resistivity phase, amplitude, and polarizability profiles. For the survey profiles, there were anomalies near the survey point No. 15, which was consistent with some independent geological and drilling information regarding the position of the target demonstrating the effectiveness of the approach.

In conclusion, this whole methodology to obtain complex resistivity and phase at multiple frequencies with high SNR consists of the following steps: 1) pseudo random signals are transmitted as the excited current, and full waveforms of the current data and potential data in multiple periods are acquired; 2) the original current data and potential data are processed by coherency analysis, in which the segments are deleted when the correlation coefficient of current and potential of these segments is less than 0.5; 3) the robust statistic method is used to stack these current and potential data from multiple periods. Then, the complex resistivity and phase are calculated using the current and potential data after stacking by the Fourier transform. Finally, the relative phase spectrum and inverse differential operation methods are used to correct the complex resistivity and phase, and the whole processing is finished.
Now, there are many electrical instrument systems with the functions of transmitting and acquiring pseudo random signals, which can be seen in these references (Chen et al., 2007; Li et al., 2013; Ilyichev and Bobrovsky, 2015). These processing methods, including coherency analysis, robust stacking, relative phase spectrum, and inverse differential operation, can be implemented through computer programming in Matlab, C++, or Fortran.

6. Conclusion

Both m-sequence $2^n$-sequence are broadband signals. By transmitting m-sequence or $2^n$-sequence current in electrical exploration, complex resistivity and phase at multiple frequencies can be obtained simultaneously with only one power supply. For the m-sequence method, complex resistivity is mainly at 1/16 Hz, 2/16 Hz, 3/16 Hz, ..., 1 Hz. EM noise is the major interference. Long-time data acquisition, coherency analysis, and robust stacking can suppress noise interference effectively, which can improve data quality and save measuring time in the
field. For the $2^t$-sequence method, complex resistivity is mainly at $2^7$ Hz, $2^6$ Hz, $2^5$ Hz, $2^6$ Hz, $2^7$ Hz. EM coupling is the major interference. At low frequencies, relative phase spectrum and inverse differential operation can suppress EM coupling interference. However, the correction is not reliable at high frequencies.

Acknowledgements. This research is supported by the special funding of fundamental research of the Chinese Academy of Geological Sciences (YYWF201632).

REFERENCES


Ilyichev P.V. and Bobrovsky V.V.; 2015: Application of pseudonoise signals in systems of active geoelectric exploration (Results of mathematical simulation and field experiments). Seismic Instrum., 51, 53-64.


Corresponding author: Lin Pin-rong
Institute of Geophysical and Geochemical Exploration, Chinese Academy of Geological Sciences, Langfang, Hebei 065000, PR China
Phone: +86 0316 2267662, fax: +86 0316 2014156, e-mail: linpinrong@126.com