Two-step data analysis for future satellite gravity field solutions: a simulation study

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Abstract. A two-step approach for gravity field recovery from future SST- or SGG-missions is discussed, where space-localizing base functions serve for modelling the anomalous field. Results of a simulation study regarding scenarios of CHAMP type (high-low GPS-SST) and GRACE type (high precision low-low SST) are presented.

1. Introduction

The task of the envisaged SST/SGG-missions like CHAMP, GRACE, and GOCE is the computation of a global gravitational field with high resolution and precision and - if possible - with repetition in time. Global recovery approaches are aimed at the computation of spherical harmonic models. But open questions are related to a violation of an ideal data coverage or, to not well-defined boundary surfaces. The global support of the spherical harmonics does not allow us to adapt precision to areas of geodynamical interest or to time-dependent phenomena.

The authors focus on a two-step approach for satellite data analysis (Fig. 1): first, the true gravitational field is approximated by space-localizing kernel functions. In combination with terrestrial data, this could supplement the satellite derived regional field information at this level. In a second step, whether the “regional” solution covers the whole earth, or independent regional solutions are merged into a global one in an appropriate way, spherical harmonic coefficients may be derived by a simple summation process.

2. Modelling

The primary observation, Eq. (1), used in this study is based on the eigenfunction expansion (Schneider 1984) of the intersatellite or gradiometry baseline Eq. (2)
These (pseudo-) observations can be computed from original SST range $r$, range-rates $\dot{r}$, or SGG differential acceleration data $\ddot{r}$ by numerical integration

$$ r_v = -2 \left( \frac{\Delta t}{\nu \pi} \right)^2 \int_0^1 \sin(\nu \pi \tau') \left( \frac{1}{r} (|\dot{r}|^2 - \dot{r}^2) + e \cdot D(V_{ref} + T) \right) d\tau', \quad (1) $$

$$ r(\tau) = \bar{r} + \sum_v r_v \sin(\nu \pi \tau), \quad (2) $$

where

$$ D_{SST} = \nabla_2 - \nabla_1, \quad D_{SGG} = r \cdot \nabla \nabla. \quad (3) $$

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$$ r_v = 2 \int_0^1 \sin(\nu \pi \tau) (r(\tau) - \bar{r}(\tau)) d\tau \quad (4) $$

$$ r_v = 2 \frac{\Delta t}{\nu \pi} \int_0^1 \cos(\nu \pi \tau) \dot{r}(\tau) d\tau \quad (5) $$
allowing a simple common processing of different data types as well as a certain degree of data compression (Ilk et al 1995). On the model side, the anomalous field $T$ is approximated using isotropic space-localizing base functions $B_j$ which unlike spherical harmonics are non-orthogonal, i.e. they possess a full Gram matrix $(B_j, B_k) = (P)_{jk}$. Noisy observations are considered as bounded linear functionals in a reproducing kernel Hilbert space $H$,

\[
    r = -2 \left( \frac{\Delta t}{\nu \pi} \right)^2 \int_0^1 \sin(v \pi \tau) \tilde{r}(\tau) d\tau,
\]

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\[
    \tilde{T} = \sum_j \chi_j B_j, \quad B_j(Q) = \sum_{n=0}^\infty \frac{2n+1}{4\pi} b_n P_n(Q, Q_j),
\]

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\[
    \dot{\nu} + \varepsilon = A^T T = (A_j, T)_H.
\]

Minimizing a weighted sum of least-squares (from data noise as well as projection error) and $H$-norm of $T$

\[
    F_{\gamma^2} = \|A_x - \hat{T}^2\|_H^2 + \gamma^2 \|\tilde{T}\|_H^2,
\]
leads to the well-known Tykhonov-regularized solution

\[ \hat{\chi} = (A^T P_{\mu} A + \gamma^2 P)^{-1} A^T P_{\mu} \hat{l} \quad (10) \]

with variance-covariance matrix

\[ \hat{C}_{\chi\chi} = \sigma^2 (A^T P_{\mu} A + \gamma^2 P)^{-1} A^T P_{\mu} A (A^T P_{\mu} A + \gamma^2 P)^{-1}. \quad (11) \]

Finally Eq. (7) implies that spherical harmonic coefficients for \( \bar{T} \) are given by

\[ \bar{T}_{nm} = (\bar{T}, Y_{nm})_{L_2(\Omega)} = b_n \sum_j \hat{\chi}_j Y_{nm}(Q_j). \quad (12) \]

### 3. Simulation study

The concept described above was verified in a simulation study regarding two idealized SST mission scenarios. Fig. 2 provides a sketch of the simulation principle.

The basic mission configurations were:

1. GRACE scenario: 2 LEO’s, baseline 300 km, 25 GPS satellites;
2. CHAMP scenario: 1 LEO, 25 GPS satellites. A LEO orbit was chosen by \( a=6732266.20 \) m, \( e=0.001, i=97.29^\circ \), mean altitude 354 km. According to the “snapshot” objective of GRACE, a mission duration of 31 days was simulated. The white noise level was \( \sigma=1 \) \( \mu \)m/s for LEO-
LEO range-rates, $\sigma=3$ cm for LEO-GPS ranges. All orbits were integrated numerically applying the EGM96 gravity field model. The area under consideration was $(\lambda, \varphi) \in [0^\circ, 10^\circ] \times [0^\circ, 90^\circ]$ with base function grid spacing of $1^\circ \times 1^\circ$.

Fig. 3 shows a) an “equatorial portion” as well as b) a “polar portion” of the resulting GRACE normal equation matrix, the greyscale corresponding to $| (A^T P_{il} A)_{ij} |$. Clearly visible is the stabilizing effect of the broadened grid spacing in the northernmost area.

Fig. 4 illustrates a) the LEO ground track pattern, b) mean residual gravity anomalies computed from the EGM96 model with reference to the low-degree model, c) the recovery result from the GRACE scenario, and d) absolute deviations. In Fig. 5 results are displayed for the idealized CHAMP mission: a) gives the $5^\circ \times 5^\circ$ mean anomalies from the EGM96 input, b) the recovery result, and c) absolute deviations.

4. Conclusions

Aiming for a spatial resolution of $1^\circ \times 1^\circ$ (100 km) in a GRACE/31 days recovery simulation,
we found an overall rms value of 6-7 mGal from the total deviations. The accuracy degrades in areas of a rough gravity field, as clearly indicated in Fig. 4d. When removing features of less than 300 km from the EGM96 anomalies by truncating the expansion at $n=120$, we found the deviations distributed randomly with an rms value of 3-4 mGal. For the CHAMP scenario we had an rms value of 4 mGal for 5° mean anomalies.

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References

