Umbria-Marche sequence (central Italy): a study on its aftershock sequence

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ABSTRACT

We studied the aftershock sequence following the mainshock ($M_s = 6.4$) of September 26, 1997 with its epicenter at Colfiorito (Italy). This work studies a methodological aspect finalized to observe possible anomalies in the temporal decay before a large aftershock with magnitude $M > 5.5$. Such anomalies are thought to be variations from the mean temporal trend. In fact, modeling the decay as a non-stationary Poissonian process, the number of aftershocks in a small time interval $\Delta t$ is the mean value $n(t) \cdot \Delta t$, with standard deviation $\sigma = \sqrt{n(t) \cdot \Delta t}$. In this work, besides the analyzed parameters of other sequences, we have extended the spectrum of parameters by using fractal properties of the temporal sequence. Earthquakes belong to a class of phenomena known as multifractals. In general, it is important to define the fractal dimension $D$, but sometimes is not useful if we are describing a natural phenomenon; so it is necessary to define the box-counting dimension $D_0$, and the correlation dimension $D_2$. Usually $D_0 \geq D_2$. The data concerning the temporal series, checked according to completeness criteria, come from the NEIC-USGS data bank (http://neic.usgs.gov/neis/epic/).

1. Introduction

On September 26, 1997 an earthquake of $M_w = 6.0$ (computed by Regional Centroid Moment Tensor www.ingv.it) occurred at Colfiorito (central Italy) at 9:40 GMT. The aftershock sequence (Umbria-Marche sequence) was closely monitored by the Istituto Nazionale di Geofisica (now Istituto Nazionale di Geofisica e Vulcanologia) through a portable network of ten stations.

Aftershocks following any major earthquake tend to cluster in space and time (Console and Di Giovambattista, 1987). Their temporal distribution often follows a regular trend, as first observed by Omori in 1894. Most of the large earthquakes that occurred in the past years (Japan 1995, Turkey, 1999, Taiwan 1999, India 2001, Indonesia 2004, etc.) had significant damage to structures and injured or killed many people.

The purpose of this paper is to identify anomalies in the temporal decay occurring in the seismic sequence before a large aftershock.

The temporal and spatial evolution of seismicity, following the mainshock in the area, has been described by Amato et al. (1998). A detailed study of the mechanism of the aftershock sequence was made by Chiaraluce et al. (1999). The tectonics of the region is affected by the Quaternary activity of normal faults oriented NW-SW. There is also evidence of a NE-SW transversal fault system, both extensional and strike slip (Deschamps et al., 1984; Ekstrom et al.,...
In this article, various features of the Colfiorito seismic sequence are taken into consideration. The Colfiorito earthquake had serious consequences, twelve people died and several buildings were seriously damaged. The sequence was developed along an approximately NNW-SSE line between Bagno di Romagna and L’Aquila. A detailed analysis of the temporal series of the sequence supports the hypothesis that it was developed by the activation of a fault segment (Nocera-Verchiano) on September 26. A large number of events in the area have a magnitude greater than 4.0. The sequence is characterized by an evident migration of seismicity (Cocco et al., 2006). The temporal and spatial evolution of seismicity following the mainshock in the area, initially, shows a cluster of activity prevalently between Nocera Umbra and Verchiano. An occurrence of another event on October 14, 1997 shows that the activity is close to Sellano (south). The aim of this study is to explain any anomaly in the temporal distribution of seismic activity, particularly by focusing on the seismic anomalies in the temporal decay of the sequence. These anomalies are addressed as seismic activity variations, in a short time interval, before the occurrence of a large aftershock. The possible explanation of this phenomenon could be found in terms of fracture of an asperity related to the large aftershock. The fracture of neighbouring asperities initiated after the mainshock is presumably due to the redistribution of the strain energy, which is released by the mainshock, resulting in enhancement of the stress concentration around the nearest neighbouring intact asperities (Lei, 2003).

2. Aftershock temporal series: some theories and models

The probability of occurrence of future large earthquakes are very interesting from the point of view of earthquake physics, and are crucial for attempts to forecast the hazards due to large, damaging earthquakes. A theoretical model that successfully describes the earthquake recurrence is unknown. So it is necessary to adapt probability distribution based on earthquake history. Usually, the frequency of occurrence of aftershocks decays rapidly according to Omori’s law (Utsu, 1961):

\[ n(t) = k / (c + t)^{-p} \]  

(1)

where \( n(t) \) is the frequency of aftershocks at time \( t \) after the mainshock; \( k, c \) and \( p \) are constants that depend on the characteristics of the area (Omori, 1894; Mats’ura, 1986). The \( p \)-value varies between 1.0-1.4. Earthquakes located within a characteristic distance from the main event can be considered as aftershocks. The spatial distribution of the aftershocks is often related to the fault area or its length. Fault distance is usually considered equal to one or two lengths of the fault segment related to the mainshock. The empirical relations between fault area \( A \) and magnitude \( M_s \) [Eq. (2)], fault length \( L \) and magnitude \( M_s \) [Eq. (3)], and size and frequency of occurrence of earthquakes [Eq. (4)] are

\[ \log A = 1.02M_s + 6.0 \]  

(2)
Here $N$ is the number of earthquakes with a fixed magnitude $M$, $a$ and $b$ are constants characteristic for the given area. The relation is valid for earthquakes occurring regionally and globally. The constant $b$ varies from region to region, but it is generally in a range of $0.8 < b < 1.2$ (Frohlich and Davis, 1993).

3. The data set

Our data set consists of 482 events with magnitude greater than 1.0 recorded from September...
26, 1997 to September 26, 1998. The data, concerning the temporal series analyzed here, come from the NEIC-USGS data bank (http://neic.usgs.gov/neis/epic). The United States Geological Survey reported the mainshock ($M=6.4$) that occurred at 43.08° N, 12.81° E. Its time of origin was 9:40 (UTC) or 11:40 local time. For our analysis, we calculated a priori the dimensions of the area using the empirical relation $\log L=0.5M-1.8$ (Utsu, 1969). We acquired data in a square with sides at a distance $3L$ from the mainshock epicenter, where $L$ represents the fault length of the mainshock computed by Eq. (3), and from the first 10 days starting from the mainshock.

Using these data, we calculated the “barycenter” of the aftershock sequence. The “barycenter” coordinates are calculated as an arithmetical mean of the latitude and longitude values of the shocks (D’Amico et al., 2004). This was introduced to better frame the aftershock epicenters, and so, to eliminate the possibility that a lot of them might remain out of the sector considered. The final, rectangular sector considered for aftershocks is centred on the sequence “barycenter” and has its sides at a distance from this point, in terms of latitude and longitude, equals to $1.5L$.

Following Caccamo et al. (2005, 2007), we computed the completeness threshold $M_c$ and the duration for the seismic sequence. The completeness threshold, $M_c$, of a data set is made using the Gutenberg-Richter (1954) relation. In this case, $M_c$ is 3.1 computed with the data related to the epicentral distribution for events with magnitude $M \geq 1$ reported by the web site of the NEIC-USGS data-bank in the first 10 days after the mainshock and, in a square centered on the barycenter (lat. 43.07° N, lon. 12.75° E). Fig. 1 shows the data used to obtain the temporal trends related to the parameters shown in Figs. 2, 3 and 4. The map is centered on the coordinates (triangle)
of the “barycenter”: 43.07° N, 12.75° E. The epicentral distribution of events with magnitude $M \geq M_c$ in a period $d=121$ days from the occurrence of the mainshock (star) is shown. The latitude, longitude and depth of events range from 42.74° N to 43.40° N, from 12.42° E to 13.08° E and from 0 km to 70 km, respectively.

Fig. 2 shows a cumulative number of aftershocks per day. It is possible to see how the sequence is stopped after ten days without shocks.

Fig. 3 shows the cumulative energy of the aftershock sequence expressed as

$$E_n = \sum_{i=1}^{n} \sum_{j=1}^{m} e_{ij} \quad n = 1, \ldots, d$$

where $e_{ij}$ represents the energy of aftershocks in the $i$-th day, $m_i$ the number of aftershocks in the $i$-th day and $n$ is the temporal length, in days, of the sequence.

The empirical magnitude-energy relation used here is

$$\log_{10} E = 1.5M + 4.2$$

with energy $E$ in Joule (Gutenberg and Richter, 1942, 1956; Udias, 1999). The energy involved in the decay process was about $9 \times 10^{13} J$. 

Fig. 3 - Cumulative energy of the aftershocks sequence. The energy involved in the decay process was about $9 \times 10^{13} J$. 


In Fig. 4, p-values are shown, where the central graph shows the p-value variations and the others represent the 95% confidence interval. The p-value may be related to the temporal trend of shocks. In fact, with an increase of the p-value, corresponds an increase of the fracturing level of the rocks which indicates a relative reduction in the stress pattern. Its value also depends on the characteristics of the area (Omori, 1894; Bottari and Caccamo, 1981; Matsu’ura, 1986; Kisslinger, 1993).

The p-value is computed by a non-linear least squares method (Dennis, 1977; More, 1977; Coleman and Li, 1994, 1996) and its computation is made during the entire temporal duration. Its value is about 0.9 at the end of the sequence. From this graph it is possible to note two minima on October 6 and 15 of 1997, respectively. The p-value decrease, preceding October 15, 1997, could indicate a resumption of activity in a quiescent area, adjoining the already active segment. This is hypothesised to indicate the possibility of a strong event.

4. Time clustering of seismicity

The fractal nature of earthquakes is widely observed. The last 20-30 years have seen a growing interest in evidence of fractal properties both in the spatial and the temporal distribution (Smalley et al., 1987; De Rubeis et al., 1993, 1997). A Laboratory experiment (Hirata et al., 1987) shows fractal distribution of rock fractures. These observations imply that a description of scaling properties of earthquakes can be carried out.

Complex phenomena often exhibit power law (fractal) scaling (Mandelbrot, 1967, 1992; Turcotte, 1997). Aki (1981) showed that the Gutenberg-Richter relation and the formula \(N = CA^{-y}\) are equivalent. Thus, the Gutenberg-Richter relation implies universal fractal behaviour of earthquakes. In the relation, \(N\) represents the number of earthquakes per unit time with rupture area greater than \(A\) occurring in a specified area; \(C\) and \(y\) are constants. The fractal dimension is \(D = 2y\) (Turcotte et al., 2000). Earthquakes belong to a class of phenomena known as multifractals. The concept of topologic dimension for any object is known. For example, a segment, a plane figure or a solid figure have, respectively, dimensions equal to 1, 2 or 3. A fractal can instead be defined as an object of fractionary dimension. To define its dimension it is important to refer to the self-similarity concept: an object is selfsimilar when it can be divided into similar parts of the whole. Fractal dimension, \(D\), is defined, using the ratio of homothety, as:

\[
D = \frac{\log N}{\log \left(\frac{1}{k}\right)}
\]  

where \(k\) is the ratio of homothety and \(N\) is the number of the selfsimilar copy obtained.

Given a volume, we divide the space into cubes of side \(\varepsilon\), and we calculate the number of occupied cubes, \(N(\varepsilon)\). The fractal dimension based on box-counting, \(D_0\), is:

\[
D_0 = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log \left(\frac{1}{\varepsilon}\right)}
\]
If we want to consider “how many of the cubes are occupied”, we must introduce another important fractal dimension, the correlation dimension $D_2$, defined as:

$$D_2 = \lim_{\varepsilon \to 0} \frac{\log C(\varepsilon)}{\log(\varepsilon)}$$  \hspace{1cm} (9)$$

where $C(\varepsilon)$ represents the “correlation sum” function defined as:

$$C(\varepsilon,q) = \sum_{i} x_i q^{i-1} = \frac{1}{N} \sum_{i} \left( \sum_{j \neq i} \theta \left( \varepsilon - |x(i) - x(j)| \right) \right) q^{i-1}$$  \hspace{1cm} (10)$$

considering the Heaviside step function:

$$\theta(x) = \begin{cases} 
1 & x \geq 0 \\
0 & x < 0 
\end{cases}$$  \hspace{1cm} (11)$$

Fig. 4 - Variations of $p$-value during the decay process for the Umbria-Marche seismic sequence on September 26, 1997. This value is related to the temporal trend of shocks. The central graph represents the parameter variations, the others represent the confidence interval at 95% level.
$N$ represents the number of points, $x(i)$ and $x(j)$ the position of various points along the axes, the absolute value represents the Euclidean distance of a pair of points that characterize the distribution and, finally, $\varepsilon$ is the length of intervals which fill the entire ensemble.

$P(i)$ represents the probability of a point belonging to the $i$-th box with size $r$. The real number $q$ is a weight that permits the investigation of more or less densely clustered parts of the set. In particular if $q=2$, $p$ represents the probability that 2 points are in the same box (Schuster, 1988).

It is possible to obtain the box-counting $D_0$ and the correlation dimension $D_2$ for $q=0$ and $q=2$, respectively. In general $D_0 \geq D_2$, when $D_0=D_2$ the phenomenon is a monofractal (Goltz, 1998). Moreover, these dimensions are less than 1 where events are clustered in time (D’Amico et al., 2005).

The temporal distribution of seismicity has been investigated by several authors using a diverse range of techniques. Time variations of the fractal dimensions of seismic sequences have been investigated, highlighting a connection with the $b$-value of the Gutenberg-Richter relation (Hirata, 1989; Main, 1991a, 1991b; Lomnitz-Adler, 1992; Oncel et al., 1996).

In the present work, the study on fractal dimensions $D_0$ and $D_2$ highlights the time clustering of seismicity related to the epicentral area of September 26, 1997 Colfiorito earthquake. This aftershock sequence has been empirically divided into four periods of interest each of which contains the events of the preceeding time intervals, expressed in days:

- period 1: 1...5;
- period 2: 1...10;
- period 3: 1...15;
- period 4: 1...20.

Fig. 5 shows fractal dimensions $D_0$ and $D_2$ calculated for each period separately. We obtained $D_0=1.0301\pm0.1930$ and $D_2=0.5372\pm0.0198$ for the first period, $D_0=0.8934\pm0.0709$ and $D_2=0.6494\pm0.0071$ for the second period, $D_0=0.8815\pm0.0413$ and $D_2=0.6480\pm0.0178$ for the third period, $D_0=0.8786\pm0.0301$ and $D_2=0.6986\pm0.0058$ for the fourth period. All the calculations have a high correlation coefficient $R$ close to 1. Fig. 6 shows the trend of $D_0$ and $D_2$ for the four periods considered. We observe that $D_2$ values appear almost constant from period 2 to period 3, while there is an increase in the first and last periods. This seems to be consistent with the occurrence of the strong aftershocks after the increase in $D_2$ values. This phenomenon should be further investigated.

5. The $\Delta/\sigma$ method

Following the approach described by Caccamo et al. (2005, 2007) and Parrillo et al. (2005), we detected the temporal decay of this aftershock sequence finding some anomalies. In mathematical terms, the observed temporal series of the aftershocks, per day, can be considered as a sum of a deterministic contribution and a stochastic contribution. These contributions are respectively the temporary decay of aftershocks with a power law, i.e. the modified Omori formula (Utsu, 1961), and the random fluctuations around a mean value represented by the above mentioned power law.

Since the decay phenomenon can be modelled as a non-stationary Poissonian process, where the intensity function is equal to $n(t)=k(t+c)^p$ (Page, 1968; Matsu’ura, 1986), the average number of aftershocks in a time interval $\Delta t$ is $n(t)\cdot \Delta t$, with a standard deviation of $\sigma = \sqrt{n(t)\cdot \Delta t}$. With
Fig. 5 - Calculations of the fractal dimensions $D_0$ and $D_2$ respectively for: Period 1 (a and b); Period 2 (c and d); Period 3 (e and f); Period 4 (g and h). These dimensions are calculated from the slope of the best straight linear the log-log plot.
Let $\Delta t = 1$ day, the mean and standard deviations become $n(t)$ and $\sqrt{n}$, respectively. We may expect that the random fluctuations around the mean value $n(t)$ will be within a $2.5\sigma$ range ($\sim 99\%$): in particular, data with $\Delta(t) \mid n_{\text{obs}} - n_{\text{calc}} \mid > 2.5\sigma$ have a probability of occurrence less than about 1% (Caccamo et al., 2005, 2007; Bussetti, 1983). We find the anomalies (the ratios of the differences between the observed temporal trend and the theoretical trend) from the best fit, and the standard deviations of the theoretical values. We considered anomalies with $\Delta / \sigma$ values greater than 2.5.

The complete series of the real data is

$$\{n_{\text{obs}}(t_j)\} \quad \text{with} \quad j=1, \ldots, d. \quad (12)$$

Hypothesizing that an anomaly occurs several days before a large aftershock, depending on its magnitude, we introduced a constant shift $s=6$. This is a starting assumption to avoid including the possible anomalies in the extrapolation (in this temporal range). The predicted theoretical value of the series, obtained by extrapolation, is

$$\{n_{\text{calc}}(t_k)\} \quad \text{with} \quad k=h+s, \ldots, d \quad (13)$$

and $h=2\nu$, where $\nu=2$ is the number of the unknown variables, $k$ and $p$, of the function we use here:

$$n(t)=k \cdot t^p + k_1. \quad (14)$$

The generic element of series (13), $n_{\text{calc}}(t_g)$, where $g=h+s, \ldots, d$, is obtained by using the subset $\{n_{\text{obs}}(t_e)\}$ with $e=1, \ldots, g-s$ (excluding the last six data from the elaboration), where $g$ is the number of time intervals according the sampling time (in this paper the sampling time is 1 day). The value “ten” is the sum of the shift ($s=6$) and the minimum number of points used for the extrapolation ($h=4$).
Fig. 7 - a) Temporal trend $n(t)$, number of aftershocks per day, for the events with a magnitude $M \geq M_c$, where $M_c=3.1$, of the Umbria-Marche sequence; b) temporal trend of the aftershocks with magnitude $M > 5.5$; c) $\Delta/\sigma$ values versus time $t$, are shown: there are 5 $\Delta/\sigma$ values, in this case, exceeding this threshold: $\Delta/\sigma=6.08$ on the 11th day, $\Delta/\sigma=5.56$ on 17th day, $\Delta/\sigma=4.79$ on the 18th day, $\Delta/\sigma=8.99$ on the 19th day, $\Delta/\sigma=6.45$ on the 20th day, with shocks with magnitude $M=5.8$ and $M=5.7$ on the 11th and 19th day, respectively.
The differences between the calculated data and the observed data, $\Delta(t_k)$, are obtained by

$$\Delta(t_k) = |n_{obs}(t_k) - n_{calc}(t_k)|. \quad (15)$$

When $\Delta(t_k) > 2.5 \cdot \sqrt{n_{obs}(j) \cdot \Delta t}$, we considered this as an “anomaly”. So, how can the above be explained, the first point of the extrapolation is relative to $t_k=10$ days.

We considered an additional constant term $K_1$ to take into account the background seismicity. This parameter is typical of the seismicity of an area. To evaluate it, we should consider several interseismical periods of the area, to estimate the background seismicity. We assumed, as a starting assumption that $K_1=1$, since the aftershock decay is a discrete phenomenon and so the function we consider here stretches asymptotically to 1 (Utsu et al., 1995; Caccamo et al., 2005, 2007).

The temporal trend $n(t)$, number of aftershocks per day, is shown in Fig. 7a. In Fig. 7b shocks with magnitude $M>5.5$ are shown. In Fig. 7c, the $\Delta/\sigma$ values versus time $t$ (in days) are shown, excluding the initial 10 days.

The three plots of Fig. 7 have the same temporal scale.

There are 5 $\Delta/\sigma$ values exceeding the threshold mentioned above: $\Delta/\sigma=6.08$ on the 11th day, $\Delta/\sigma=5.56$ on 17th day, $\Delta/\sigma=4.79$ on the 18th day, $\Delta/\sigma=8.99$ on the 19th day, $\Delta/\sigma=6.45$ on the 20th day, with shocks with magnitude $M=5.8$ and $M=5.7$ on the 11th and 19th day, respectively. From Fig. 7, it can be seen that the first anomaly is contemporary to the October 6 ($M=5.8$) aftershock, while the other ones occurred before the October 14 large aftershock, except the last $\Delta/\sigma$ value, that we consider as a false alarm. We identify a correlation between the $\Delta/\sigma$ anomalies and the occurrence of the large aftershocks.

6. Discussions and conclusions

The aim of this work is the identification of anomalies in the temporal trend occurring in the seismic sequences before a large aftershock. These anomalies are thought to be variations in seismic activity, in a short time interval, before the occurrence of a large aftershock. It is observed that, before the occurrence of the large aftershock, it is possible to indicate these kinds of anomalies in the temporal decay, considered as deviations from the mean value $n(t)$ greater than a 2.5 quantity, that, likely, are not random-like fluctuations but a possible precursor.

The most sensitive parameter for the individuation of seismic anomalies of this aftershock series is the $\Delta/\sigma$ value relative to the temporal trend $n(t)$. The other parameters we use for completeness of information, do not furnish, for this purpose, significant results.

Within the previous approximations, it seems that these fluctuations are consistent with the occurrence of the large aftershocks (Fig. 7b) and the trend of $D_0$ and $D_2$ values (Fig. 6) too.
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