MULTIFRACTAL DISTRIBUTION OF SEISMIC MOMENT

Abstract. The conventional approach to seismicity stated that in any given earthquake prone area, the main events are generally randomly distributed in time. However it is now also well established that in some situations earthquakes are clustered both in space and time. Using the global catalog of earthquakes with magnitude $M_W \geq 6$, we show that the seismic moment release is characterized by a global multifractal clustering.

INTRODUCTION

Seismicity represents a clear example of a complex spatio-temporal phenomena (see e.g. Turcotte, 1993; Sornette, 1999 and references therein). Even if some experimental problems are present in recording earthquakes or understanding what kind of data we accumulate, the obvious general interest in forecasting earthquakes gives us the unique possibility to have at disposal a lot of different data sets, with different degree of completeness. Very often the main events mark the beginning of a series of aftershocks, and, in some cases, they are also preceded by foreshocks. Both foreshocks and aftershocks are generally well space-correlated, their sources being distributed along or near the same fracture surface of the main shock. On the contrary the conventional approach to seismicity stated that main shocks are generally uncorrelated in time. Recently people convinced themselves that clusters are present even in the sporadic shocks. Then, in order to investigate the statistics of earthquakes mainly in connection to the problem of seismic hazard, the statistical approach tends to eliminate clusters.

Recently the statistics of global seismicity has been investigated without distinguishing between foreshocks, aftershocks and sporadic shocks (Sotolongo-Costa et al., 2000; Bak et al., 2002; Sornette and Helmstetter, 2002; Helmstetter, 2003; Mega et al., 2003). As a matter of fact there is no unique operational way to distinguish between these events. Then, as a new approach, we would like to investigate global seismicity in the framework of a new idea that comes into play to understand the statistic of main shocks. Seismicity could be viewed as the result of a scaling process where clusters are presents at all dynamically interesting scales, even in the main shocks. If this approach would results physically justified, the
tentative of eliminating clusters does not make sense. In this framework seismicity will be characterized by the global scaling laws of clusters rather than by the mixing of stochastic main events and correlated aftershocks. We would like to point out the exclusive character of both approach. In the conventional approach the presence of clusters at the largest scales is an event with a very low probability of occurrence, so that de-clustering is justified. According to the other approach, the presence of clusters at all scales, even if with different probabilities, is one of the main statistical law by which we get information on the global physical process which generates seismicity. In this last perspective what is interesting to be investigated are the clustering properties.

DATA ANALYSIS

To work out the above idea and investigate clustering within seismicity, we considered a global catalogue based on the Centroid Moment Tensor (CMT), catalogue – Harvard (http://www.seismology.harvard.edu/CMTsearch.html). In particular, for sake of for the period 1984-2002 completeness, we consider events with magnitude $M_W \geq 6.0$.

![Fig. 1 - Time point process of seismic moment $M_0(t)$ for the global seismicity.](image)

In Fig. 1 we report the time evolution of the seismic moment $M_0(t)$. As a reference the reader can note the striking analogy between our Fig. 1 and the Fig. 1 in Meneveau and Sreenivasan (1991) showing the typical time evolution of the one-dimensional energy dissipation rate in a turbulent flow. The seismic moment release shows spikes at apparently random times, the same point process can be evidenced in the time evolution of turbulent dissipation owing to the phenomenon of intermittency in fully developed turbulence. This behaviour can be thought of as representing the near-singular characteristics of the phenomenon.
To see this we perform a scaling analysis of \( M(t) \). For each time interval \( \Delta t \) we define a probability measure by dividing the time range into disjoint subsets \( \Omega_i \). The measure \( P_i(\Delta t) \) is defined as the total seismic moment in the \( i \)-th subset characterized by a time scale \( \Delta t \), normalized to the total seismic moment in the data set. This can be related to the probability of occurrence of a certain amount of energy released in the \( i \)-th box over a time interval \( \Delta t \). In Fig. 2 we show \( P_i(\Delta t) \) for different scales \( \Delta t \), as a function of time. As can be noted the general behaviour of the measure on a given scale appears to be similar to that on any other scale. What is interesting is that strong recurrent peaks at the same time are systematically detected on all scales \( \Delta t \). This indicates that seismic moment is highly localized in time with a strong scale-independent probability. These structures are now identified as the signature of clusters of seismicity at a given scale. Of course even when we eliminate the main clusters, other clusters are still present, observed at different scales.

![Fig. 2 - Time evolution of the probability density \( P_i(\Delta t) \) at the different scales \( \Delta t \) reported on the figure.](image)

Following Gefen (1981), ordered fractal models have been helpful for understanding the interplay between the geometrical structure of a cluster and its physical properties. In particular such models allowed exact evaluations of a multifractal spectrum. In fact in most interesting practical situations like that at hand, we are dealing with random fractal structures, that is the self-similarity apparently seen in Fig. 2 is not global. Rather seismic moment exhibits statistical (or local) self-similarity. Multifractal was first introduced by Mandelbrot (1974), and represents infinite sets of exponents which describe the power law scaling of all the moments of a distribution of some quantities which are defined on a fractal structure. In many cases specific members of these families of exponents coincide with the fractal dimensionalities of geometrical substructures of the underlying fractal.

Although knowledge of the multifractal spectrum is completely equivalent to knowledge of the corresponding probability distribution, the literature contains many attempts to attach a much deeper significance to the former. Then, following Halsey et al. (1986), to define the multifractal spectrum of the probability measure we use the box-counting method by defining the generalized partition functions

\[
\chi^{(q)}(\Delta t) = \sum_i [P_i(\Delta t)]^q
\] (1)
where the sum is extended to all subsets $\Omega_i$ at a given scale $\Delta t$. Higher values of $q$ enhance the strongest singularities, say the most intense clusters of the seismic moment, while negative values of $q$ in the partition function emphasizes intervals of time where the probability measure $P_i(\Delta t)$ is low. The information relative to the multifractal structure can be recognized by calculating the generalized Rényi dimensions $D_q$ from the scaling relation

$$\chi^{(q)}(\Delta t) \approx \Delta t^{(q-1)D_q}$$

(2)

For a mono-fractal $D_q$ is constant with respect to $q$. By contrast if the clustering of seismicity is described by multifractality we would find $D_p < D_q$ for $p > q$. This means that higher order clusters lies on sets of lower dimension, or which is the same, the probability of occurrence of high-order clusters decreases with respect to low-order clusters. As an example $D_0$ is the dimension of the support of the measure which indicates how well earthquakes fill the monitored time interval. Earthquakes concentrate asymptotically on a set of dimension $D_1$, while $D_2$ is the correlation dimension of the measure (Grassberger and Procaccia, 1983), which represents the usual fractal dimension calculated when the structure is approximated as a mono-fractal.

![Fig. 3 - The multifractal spectrum (q-1)$D_q$ versus q in the interval from 0.1 to 5.1.](image)

We have computed the partition functions (2) of the seismic momentum shown in Fig. 1, for 50 values of $q$ in the interval from 0.1 to 5.1. In Fig. 3 we show the behaviour of $(q-1)D_q$ versus $q$, as obtained by fitting the power-law part of Eq. (2).

The multifractal signature $D_q \neq constant$ is evident. In particular, the decreasing character of $D_q$ for increasing $q$ indicates the presence of clusterization. As for the first dimensions already mentioned, we obtained $D_0=0.96$, and $D_2=0.85$. The first result implies that our dataset is filling the observation interval quite well. The second
one shows that the approximating fractal dimension of the seismic moment support is less than the topological dimension 1.

**CONCLUSIONS**

Summarizing, we have shown that the global seismic moment owes temporal multifractal properties. This has been obtained by the analysis the topological features of the probability measure through the partition functions scaling laws. This result indicates the presence of clusterization at all temporal scales, and thus suggests the validity of the approach discussed above. In this view, earthquakes should be considered as events characterized by scaling properties at all scales. Thus, sporadic events are not described by a stochastic process, but have the tendency to clusterization, as described by their multifractal probability measure.

**REFERENCES**


Turcotte D.L., 1993: *Fractals and chaos in geology and geophysics*. Cambridge Univ. Press