CORRELATION BETWEEN PARAMETERS OF THE AFTERSHOCK TIME-MAGNITUDE DISTRIBUTION: INFERENCES ON THE PRODUCTIVITY OF THE SEQUENCES

Most current approaches to aftershock rate modeling and forecasting refer to scaling law that were empirically found to well fit the data, like the frequency-magnitude Gutenberg and Richter (1944) (now on G&R) law and the Modified Omori Model (Utsu, 1971) (now on MOM) for rate decay with time. Reasenberg and Jones (1989) proposed a simple comprehensive model to describe the aftershock occurrence as a non-stationary Poisson process whose rate varies with time $t$ after the main shock according to

$$\lambda(t) = \frac{10^{a+b(M_m-M_m^0)}}{(t+c)^p} \quad (1)$$

In the previous expressions, $bM_{min}$ represents the term accounting for the dependence of the number of shocks on minimum magnitude according to the G&R law, while $a+bM_m$ corresponds to the G&R intercept of the considered dataset and can be seen as the global aftershock productivity of the given sequence. In the latter term, first addend represents the magnitude independent productivity while second one accounts for the productivity factor related to main shock magnitude. The assumption made by Reasenberg and Jones (1989) that the linear coefficient of $M_m$ is equal to $b$ has to be considered only as a tentative hypothesis since it cannot be demonstrated by theoretical or empirical arguments.

A purely empirical approach to this question can derive from statistical considerations: if the magnitude-dependent productivity term $bM_m$ well describes the dependence of Log $N$ on main shock magnitude we would expect that it is not correlated with the magnitude independent productivity $a$. If instead such a correlation actually does exist, this can be the symptom of an incorrect formulation.

In our present analysis we consider the estimates made by Eberhart-Phillips (1998) for New Zealand, Lolli and Gasperini (2003) for Italy and Reasenberg and Jones (1989) for California. We evaluated the existence of linear correlation among couples of variables, for the above datasets, by computing the linear correlation coefficient and estimating its statistical significance.

A weak but significant positive correlation can be observed between parameters $a$ and $p$ for the merged data set including sequences both from Italy (Lolli and Gasperini, 2003) and New Zealand (Eberhart-Phillips, 1998). A similar correlation can also be found between parameters $a$ and $c$. Although relatively weak, the correlations found can be ascribed to the effect of the implicit inclusion in parameter $a$ of the time dependence normalization integral

$$\text{Int} = \int_0^t (t+c)^p dt \quad (2)$$
In fact, if we consider instead parameter \( a_0 = a + \log_{10}(\text{Int}) \), that is free from this contamination, the correlation disappears with respect either to \( p \), \( c \) and \( \log c \).

A similar analysis shows that parameters \( a \) is significantly correlated with \( b \). As argued above, this might be the symptom of the inappropriateness of assuming the coefficient of the main-shock magnitude dependent productivity equal to \( b \). On the contrary, the absence of correlation between \( a \) and \( b\text{Mmin} \) confirms that the latter term well accounts for the cutting of earthquakes below minimum magnitude according to the G&R law. From these evidences we could infer that parameters \( a \) is biased by the inappropriate choice of the \( Mm \) coefficient.

The scatter-plot analysis of \( a \) versus \( b\text{Mm} \), computed separately for the three regional datasets, indicate highly significant correlation in all cases. The estimated values of regression coefficient \( g \) are quite similar among the different datasets, ranging from \(-0.36\) (for New Zealand) to \(-0.35\) (for Italy and California). We can thus define empirically two new parameters as

\[
\begin{align*}
b_1 &= (1.0 + g)b \\
a_1 &= a + (b - b_1)Mm = a - gMm
\end{align*}
\]

So that the Log\( N \) equation would become

\[
\log N = a_1 + b_1 Mm - b\text{Mmin}
\]

where \( a_1 \) is now obviously uncorrelated to \( b_1\text{Mm} \) and even to \( b \).

Unfortunately eq. (4) cannot be simply substituted in eq. (1) to estimate parameters values from the data of single sequences due to the interplay between parameters \( a_1 \) and \( b_1 \). One possible solution is to impose \( b_1 \approx 2/3b \) and compute \( a_1 \) by common maximum likelihood techniques. We found in this case that the average values of \( a_1 \) for the three dataset as well as for the merged dataset are very close to 0 (0.023 for California, \(-0.123 \) for Italy, 0.378 for New Zealand and 0.039 for the merged set). So a possible alternative is to fix \( a_1 = 0 \) and remove such parameter from equation (1) then estimate a different \( b_1 \) for each sequence.

Different from the results obtained for Japan by Guo and Ogata (1997) we do not find a significant correlation between parameters \( p \) and \( b \) for any of the analyzed datasets. For Italy and New Zealand we find instead a clear positive correlation between \( p \) and \( \log c \) (for California the analysis is not significant as \( c \) have been fixed to 0.05 for all sequences). This might indicate an inadequacy of the MOM itself in describing the real properties of the sequences.

REFERENCES