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THE ROLE OF THE INTERFERENCE POLYNOMIAL IN THE EULER DECONVOLUTION ALGORITHM

The Euler deconvolution method belongs to the most discussed interpretation methods in potential fields during last several years. It uses in an effective way the Euler homogeneity theorem, which can be applied on functions, describing the anomalous potential field (gravitational, magnetic) of simple sources. The interpretation equation can be written in the form (Thompson, 1982):

\[(x - x_0) \frac{\partial f}{\partial x} + (y - y_0) \frac{\partial f}{\partial y} + (z - z_0) \frac{\partial f}{\partial z} = -N(f - B)\]  

(1)

where \(x, y, z\) are the coordinates of the point, where the potential function \(f\) (potential or its higher derivatives) is defined; \(x_0, y_0, z_0\) are the coordinates of the source; \(N\) is the so called structural index (sometimes the abbreviation SI is used) and \(B\) is the so called "background" term, describing the constant contribution of the regional field. The role of the value \(N\) during the application of the method is very important, it describes the type of the source, which contribution is recognised in the interpreted data. Thompson (1982) defined it as a measure of the rate of change with distance of the potential function – but this property definition holds only for point or line sources, which will be discussed later in this text. The values of \(N\) where derived and published by various authors (e. g.: Thompson, 1982; Reid et al., 1990; Stavrev, 1997; Yaghoobian et al., 1992), here a short summarisation of the most used sources is presented.

<table>
<thead>
<tr>
<th>Elementary body model (source type)</th>
<th>Number of infinite dimensions</th>
<th>(N) used in magnetometry</th>
<th>(N) used in gravimetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Pipe (vertical cylinder)</td>
<td>1 (z)</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Horizontal cylinder</td>
<td>1 (x-y)</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Dyke (sheet)</td>
<td>2 (z and x-y)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Sill</td>
<td>2 (x and y)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>contact</td>
<td>3 (x, y, and z)</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

From the presented values it follows that there exist a relationship between the value \(N\) and the number of infinite dimensions of the model body and there exist a constant shift between the values used for the magnetic and gravity fields. These properties were generalised by Stavrev (1997). The negative value of the index for the contact model in gravimetry comes directly from this generalisation and it may look little bit strange in the light of the Thompson’s definition. It would express a fact that the field is growing with the distance from the body. This is in my opinion a “trap” of this mentioned definition. When we are really rigorous, we have to mention that also the sill and dyke models have a strange index – zero (which would describe a situation, when the field is not changing with the distance). The Thompson’s
definition holds only for functions \( f \), which can be described by rational functions of type \( 1/(r^N) \) - a pole, dipole and lines of poles and dipoles (sphere, vertical and horizontal cylinder), as it was mentioned in the begin of this text. For the expression of the direct problem of more complicated bodies (bodies of a sheet form bodies, step, contact) we use functions of type \( \arctan() \) and/or \( \ln() \). It is very interesting that the derivations of the \( N \) value for these bodies give in general values of 0 or even -1. So in the scope of these facts, it is more correct to speak about \( N \) only as a homogeneity degree.

Here it is very important to mention that the gravity field of the contact model is not strictly homogenous in the sense of the Euler theorem (Pasteka, 2001). But the method can be applied to the gravity data in a modified way – with an introduced interference polynomial to the right-hand side of the interpretation equation (Eq. 1):

\[
(x - x_0) \frac{\partial f}{\partial x} + (y - y_0) \frac{\partial f}{\partial y} + (z - z_0) \frac{\partial f}{\partial z} = -Nf + A_0 + A_1x + A_2y + A_3xy + A_4x^2 + A_5y^2 + \ldots (2)
\]

where \( A_0, A_1, \ldots \) are coefficients of the interference polynomial.

The idea of the introduction of this polynomial was adopted from the well known Werner deconvolution (Werner, 1953; Hartman et al., 1971) and the first aim was to suppress the interference of neighbouring anomalies (Fig. 1). In comparison with the Werner deconvolution, in the case of Euler deconvolution the polynomial was added only to the right side of the interpretation equation.

Fig. 1 - Comparison of Euler depth estimates for the modeled magnetic field of three dykes (used structural index \( N = 1 \)) using the algorithm with and without the interference polynomial. Solution with an adopted polynomial of 2. degree give better focused cluster of estimates and also the depth information is more correct (e.g. in the case of the right-hand side edge of the central dyke). The size of the used moving window was equal for both applications = 15 profile points.

Later on, during the realisation of model studies it was realised that when we use the negative value \( N = -1 \) for the recognition of the contact or step structure in
gravity data, we have to adopt the interference polynomial with higher orders (from model studies it follows that the second order degree gives best results). This fact is displayed in the Fig. 2, where are shown results from the application of the Euler algorithm with various degrees of the adopted interference polynomial (0., 1. and 2. degree). It can be clearly seen that the correct focusing of the solutions is completed, when we adopt the interference polynomial of 2. degree (for higher degrees the solutions became little bit instable and defocused – not shown on Fig. 2).

Fig. 2 - Euler depth estimates for the modeled gravity field of an inclined contact (used structural index \( N = -1 \)) using the algorithm with and without the interference polynomial. The solution obtained without with the interf. polynomial create serious “artifacts” and the position of a small cluster is completely wrong (approx. 38 m). The solution obtained during the implementation of an interf. polynomial of 1. The best solution were obtained during application of the polynomial of 2. degree – solution are focused in a cluster very close to the real position of the source (with an error of approx. 10% - they are deeper). The size of the used moving window was equal for all applications = 25 profile points.

The form of the interpretation equation with the interference polynomial unifies various used forms of this equation: the first simple version without the “background” \( B \) term on the right side (e.g. Hood, 1965), where \( A_0 = A_1 = \ldots = 0 \); the Thompson’s version with the \( B \) term (Eq. 2), where \( A_0 = NB \) and \( A_1 = A_2 = \ldots = 0 \) and in the end also the modified equation for the contact structure in magnetometry (Reid et al., 1990):

\[
(x - x_0) \frac{\partial f}{\partial x} + (y - y_0) \frac{\partial f}{\partial y} + (z - z_0) \frac{\partial f}{\partial z} = A
\]  

(3)

where \( A \) is a constant, incorporating amplitude, strike and dip factors. Here we can define by comparing with Eq. 2: \( A_0 = A, A_1 = A_2 = \ldots = 0 \) and of course \( N = 0 \).

The very important fact, occurring during the generalisation of the Euler deconvolution algorithm by means of the introduction of the interference polynomial
is unfortunately a growth of the instability of the interpretation equation (Eq. 3). Simple tests with Gaussian normal noise, added to the synthetic data (Pasteka, 2004) have showed that a very small level of it (only 1%) have caused a complete failure of the method – not one usable solution could be obtained without preliminary smoothing of the input data. Application to real data follows the same way – very typical is an existence of a large amount on erroneous solutions. A more detailed analysis of this instability and search for an improvement by means of regularisation techniques is performed in present days with the aim to describe a stabile algorithm for the use in the practice.

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