Introduction. The scattering of seismic waves propagating in heterogeneous media is more complex than the scattering of other waves (electromagnetic, acoustic) due to the conversion from P to S and vice versa. The multiple scattering theory describes sufficiently well the energy decay along the earthquake coda. When the travel time is much greater than the mean free scattering time and the travel distance is much greater than the mean free scattering distance, the seismic wavefield is expected to be diffuse, as observed for electromagnetic and acoustic waves propagating through heterogeneous media. When a high number of waves of similar amplitude reach the observation point at the same time with different propagation directions and random phase, the resulting wavefield is said diffusive. The most relevant features of such wavefield are a small variation of amplitude with time and a low coherence. Regarding seismic waves, a wavefield is diffusive at a given frequency if in any chosen time window there are many P and S waves (and surface waves if the observation point is at or near the free surface) of comparable amplitude propagating with different random directions and random
phases. A signal like that is expected to have a low coherence. Seismic noise usually has the characteristics of a diffusive wavefield.

The earthquake source radiates energy in a broad frequency range, therefore the wavelength of seismic waves involved in the scattering process cover a broad range of distance. Moreover, the scattering coefficient $Q_s$ has been observed dependent on the frequency in most of studied regions. For these reasons, a seismic wavefield may be diffusive at a given frequency but not at other frequencies. Higher frequency waves are expected to reach a diffusive regime before lower frequency waves. On the other hand, higher frequency signals attenuate faster than lower frequencies, therefore the late coda spectrum is dominated by low frequency energy. This means that diffusive wavefield of seismic waves may not be observed at any frequencies in the same time window along the late coda.

Theoretical arguments about the multiple scattering imply the energy equipartition of a diffusive wavefield. In a diffusive regime the available energy is equally distributed, in fixed average amounts, among all the possible states (Sanchez-Sesma et al., 2011). Another way to state the equipartition principle is that the phase space is uniformly populated (Weaver, 1982). Regarding seismic waves, the properties of conversion from P to S and from S to P are such that at the equilibrium (diffusive regime) the following energy relation holds (Papanicolaou et al., 1996; Hennino et al., 2001):

$$\frac{E_S}{E_P} = 2 \frac{\alpha^3}{\beta^3} \approx 10$$

From this relation we expect S waves to be much more abundant than P waves along the earthquake coda. The few attempts to measure energy partition in the earthquake coda have given controversial results, mostly because separating P and S waves in a diffusive wavefield is impossible. The fact that most of seismic stations are installed at surface, or at most at few hundreds meters depth, and the strong heterogeneity that characterize the shallowest layers further contribute to the results uncertainty. In this paper we take advantage of a seismic array installed underground, at about 1.4 km depth, in the INFN Laboratory of Gran Sasso (Italy), from 2003 to 2010. The data from this array (named Underseis, hereafter UND) offer several advantages compared with typical surface single station recordings. First of all, the depth of 1.4 km makes negligible the contribution of surface waves for frequency greater than about 2 Hz. Second, the high number of stations in a small area permits to measure the spatial coherence of the wavefield. Third, the analysis by array methods gives an estimation of the propagation properties of the seismic wavefield. Fourth, site effects at the array stations are negligible because all stations are installed in the same geologic environment, constituted of hard rock limestone (Catalano et al., 1984).

**Methods of analysis.** The coherence of the seismic wavefield among the array stations is estimated by applying the equation:

$$\gamma(\omega) = \sum_{i \neq j} \frac{|c_{ij}(\omega)|}{\sqrt{|c_{ii}(\omega)| |c_{jj}(\omega)|}}$$

(Foster and Guinzy, 1967), where $i$ and $j$ represent the stations, and $c_{ij}$ is the smoothed cross spectral matrix defined as:

$$c_{ij}(\omega) = \sum_{k=-N}^{N} a_k F^*_{i}(\omega + k \Delta \omega) F_{j}(\omega + k \Delta \omega)$$
\( F_i(\omega) \) is the discrete Fourier transform of the \( ith \) signal, and \( a_k \) are smoothing coefficients chosen in the range 0 – 1. It is noteworthy that without smoothing \( c_{ij}(\omega) \) would be always equal to 1 (Bath, 1984). The coherence gives a measure of the signal similarity among the array stations as a function of frequency. It can be computed between two signal windows recorded at the same place but different times (time coherence), or between two signals recorded at the same time at different places (spatial coherence). In the first case the coherence measures the persistence of similarity in time, while in the second case it measures the persistence of similarity as the signal propagates from a place to another. The case of coherence between signals recorded at different places and different times is not of interest in seismology, and will not be deepened any more. On the contrary, we will focus our attention on the spatial coherence.

Many methods of analysis are available to study the propagation properties of the seismic wavefield, in both frequency and time domain. All of them, applied to time window sliding along the signal, furnish azimuth and apparent velocity of the most coherent signal. In this work we have applied the Beam Forming method (BF) in frequency domain, and the Semblance method (Semb) in time domain.

The partition of energy is evaluated in frequency domain by calculating power spectra of three component seismic signals as described by Nakahara and Margerin (2011). The energy partition ratio \( PE_{hr} \) and \( PE_{ud} \) for horizontal and vertical components) and \( H/V \) spectral ratios on coda waves are calculated on 20 s time windows starting after twice S-wave travel time and are given by:

\[
H/V = (N + E) / 2V
\]

\[
PE_{hr} = (N^2 + E^2) / (N^2 + E^2 + V^2)
\]

\[
PE_{ud} = V^2 / (N^2 + E^2 + V^2)
\]

where \( E, N \) and \( V \) are the amplitude spectra of east-west, north-south and vertical components of motion respectively.

**Data analysis and results.** UND array was composed of 20 short period three component seismic stations installed at an average distance of about 90 m for a maximum extension of about 0.5 km (Scarpa et al., 2004; Saccorotti et al., 2006; Formisano et al., 2012). The wavelength well sampled in space by the array is in the range \([0.1 \text{ km}, 0.5 \text{ km}]\), which corresponds to frequency in the range 10 Hz - 50 Hz for P waves, and 5.7 Hz - 30 Hz for S waves, assuming a P wave speed of 5 km/s. Since data were acquired at 100 sps, the intrinsic high frequency limit of our signals is 40 Hz. However, many stations were affected by local sources of noise, particularly important in many cases in the band 23 Hz – 40 Hz. Therefore we trust that our data set is suitable for measuring the spatial coherence in the 5 – 23 Hz range. At lower frequency we expect the signals be more coherent as the wavelength increases, even for diffusive regime and seismic noise, because the array extension is smaller than the signal wavelength.

We selected many local and regional earthquakes characterized by high signal to noise ratio (SNR). They were analyzed by applying array methods to many different frequency bands in the range 1 Hz – 20 Hz. Since the array consisted of three component stations, the array methods were applied to the three components independently. Beside the array analysis, the coherence of seismic signals among the array stations was computed on sliding window of length 10 s. Results have been plotted by colors versus time and frequency. Fig. 1 shows an example of this analysis for a regional earthquake. The spectrogram is also shown in the same picture to give a precise idea of the coda decay. The coherence is characterized by very high values (near 1) for signals at frequency lower than 1 Hz in the seismic noise and along the earthquake coda. For the P wave and its early coda the coherence takes the maximum at
frequency in the range 2 – 18 Hz. On the contrary, for this earthquake the coherence of the S wave and early coda is very high only at frequency lower than 3 Hz. The very high coherence of any signals at low frequency is a consequence of the array extension, which is smaller than the signal wavelength for frequency lower than about 6 Hz (S wave). In other words, UND array is not appropriate to measure the spatial coherence of seismic waves at frequency smaller than about 3 Hz. This consideration is also independent from the window length. In fact, whatever the window length is, the low frequency signals will be always very similar to each other.

Fig. 1 – Regional earthquake 200909200350. a) Seismograms of one station, Spectrogram of the vertical component and Coherence among the array stations. b) Results of the array analysis with the Beam Forming method focused at 6 Hz: Coherence at 6 Hz, spectral amplitude, backazimuth and slowness. Different symbol and color refer to the three components of ground motion.
other at the array stations, independent of their origin (earthquake, body waves, coda waves, seismic noise) and their regime (diffusive or not). At frequency higher than 3 Hz the coherence varies greatly between seismic noise, earthquake body waves, and along the coda. In this range it is noteworthy the strong similarity between spectrogram and coherence patterns. The high coherence line between 23 and 25 Hz reveals a coherent noise produced inside the laboratory. Looking at the pictures of Fig. 1 we can choose the best windows along the coda for successive analysis. Our purpose is to verify the hypothesis that the late coda is composed by a diffuse wavefield. Our data set allows the investigation of this feature in the frequency range 5 – 23 Hz.

Results of BF analysis for the same regional earthquake are also shown in Fig. 1b by different symbol and color for the three components of ground motion. The second plot contains the coherence, shown by symbol and referred to the left axis, and the spectral amplitude, shown by lines and referred to the right axis (with log scale). The coherence shown here corresponds to the same values readable at frequency of 6 Hz in the coherence of the third plot of Fig. 1a. The spectral amplitude suggests that the earthquake coda at 6 Hz lasts until at least 240 s, perhaps more. Coherence returns to values comparable to the seismic noise preceding the earthquake at about the same time. Important indications about the end of coda come also from the values of backazimuth, that return comparable to the seismic noise after 220 s, and from the slowness, that suggests 250 s. From the earthquake onset (30 s) to 260 s the values of slowness are always smaller than 0.35 s/km, corresponding to 3 km/s of apparent velocity. This demonstrate that the earthquake seismic wavefield recorded at the underground array is composed only of body waves at the frequency of 6 Hz, as expected. The slowness distributions obtained from the analysis performed at different frequencies confirm that surface waves are negligible in the signals recorded at UND array for frequency greater than 3 Hz as also shown by La Rocca et al. (2013).

In a wavefield composed of only body waves in diffusive regime (far from the surface) the horizontal to vertical ratio \( \frac{H}{V} \) must be very near to 1. Similarly, the \( PE_{hr} \) should be very close to 0.67 (Hennino et al., 2001). Therefore we have computed the \( \frac{H}{V} \) ratio and \( PE_{hr} \) along the coda of local and regional earthquakes. To perform the spectral analysis, the time window along the coda has been chosen looking at the values of coherence and spectral amplitude. We require low coherence and spectral amplitude at least 10 times that of the seismic noise preceding the earthquake. The low coherence is a condition required by the diffusive regime,

Fig. 2 – Energy partitioning ratio of two horizontal components \( PE_{hr} \) and Horizontal over Vertical amplitude ratio \( \frac{H}{V} \) versus frequency computed as the average among the array stations for the earthquake 200909200350. The different colors (red, black and blue) are referred to time and frequency limits (respectively [0-5], [5-10] and [10-22] Hz) shown by boxes in Fig. 1a. The green and magenta lines show the expected value for \( \frac{H}{V} \) ratio equal to unity (top) and equipartition in an homogeneous halfspace far from free surface (bottom) respectively.
while the high SNR condition is necessary to make negligible the effects of the many local sources of noise active in the underground laboratory. Looking at Fig. 1 we see immediately that there is not any windows along the coda where the two conditions are verified in the whole frequency range we are studying. Therefore it is necessary to split the frequency range in different intervals that will correspond to different time windows along the coda. The boxes in spectrogram and coherence of Fig. 1 shows as an example the time windows and their corresponding frequency intervals adopted for that earthquake. After having selected the time window, Fourier spectra are computed for each component of any stations over a 20 s sliding window with 50% overlapping. The spectral amplitude ratio $H/V$ and spectral power amplitude ratio are computed as a function of frequency, and finally the mean and standard deviation among the array stations are computed. At this point we consider only the result in the frequency interval corresponding to the selected time-frequency window. The result for a regional earthquake is shown in Fig. 2. The spectral ratios in Fig. 2 show at least two distinct trends in two different frequency band of analysis:

- $H/V$ is very close to unity (equal to 1.1 - 1.2) and $PE_{hr}$ is equal to 0.67 within error bars in the high/medium frequency range (from 7 to 22 Hz) identified by blue and partly black lines;
- for the low frequency range (0-7 Hz, mostly in red), $H/V$ depart from a constant value with oscillations around unity and $PE_{hr}$ assumes values between 0.4 and 0.8.

The results for high frequencies band (7-22 Hz) is a clue of equipartioned coda in subsurface under the hypothesis of wavefield composed by body waves propagating in an homogeneous medium.

Further observations can be performed for low frequencies part of energy partition by simulating energy ratios and comparing them with the observed ones (Margerin et al., 2009). By considering a reliable velocity and density model for the area under investigation (Capuano et al., 1998; Chiarabba et al., 2009), energy partition ratios have been simulated by applying a method based on a spectral decomposition of the elastic wave in a stratified half-space (Margerin, 2009). The energy ratios have been calculated at different depths (between 1 and 2 km) and for three different assumptions on the origin of coda waves: 1) coda composed of body waves incident from below the crust only; 2) coda composed of surface and guided waves in the crust only; 3) the coda is the sum of the two previous contributions. The best qualitative agreement is obtained for 1.4 km depth (green line).

Fig. 3 – Observed (red line with error band) and simulated values of $PE_{hr}$ (lines with other colors) obtained for different depths (see legend) and under the hypothesis of coda waves composed by surface and body waves. The best qualitative agreement is obtained for 1.4 km depth (green line).
agreement is obtained under the hypothesis n. 3 (body and surface waves) as shown in Fig. 3, where the red line with errors are the observed $PE_{hr}$ while the curves with other colors show the simulations for different depths. The best agreement is obtained for 1.4 km depth which coincides with the depth from surface at which UND array was located.

**Discussion and conclusion.** The results of our analysis on observed coda waves (Coherence, propagation parameters, H/V ratio) indicate that the late coda of local and regional earthquakes recorded at about 1.4 km depth is composed of body waves in a diffusive regime for frequency greater than 3 Hz. As the lapse time increases along the coda the features of diffuse wavefield become more evident. This is particularly evident for regional earthquakes. For local earthquakes the observations are limited by the coda duration.

The energy equipartition in the late coda is inferred by the results of $PE_{hr}$. $PE_{hr}$ has been calculated in different coda windows for different frequency bands in order to assure the conditions of diffuse wavefield (time windows starting after twice S-wave travel time, low coherence and spectral amplitude of coda waves higher than noise). For the frequency band 8-20 Hz the observed $PE_{hr}$ is very close inside the errors to the expected value for equipartition in an homogeneous halfspace. However, a final proof of the energy equipartition would be achievable only by using a 3D underground array. That would allow for a complete separation of longitudinal and shear waves.

By considering a stratified halfspace we have simulated energy partition in the case of diffusive wavefield and simulated and observed energy partition curves show a qualitative agreement in the frequency range 0.5 - 5 Hz under the hypothesis that coda are composed by surface and body waves. We believe that the main cause of discrepancy between observed and simulated results is due to the flat layer model that does not fit the area of Gran Sasso massif. In future simulations it will be necessary to include the local topography.

**References**


