THE IMPACT OF RAYLEIGH WAVES ELLIPTICITY IN MODE MISIDENTIFICATION

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Introduction. The surface wave method is widely diffused for shear wave velocity estimation. It is based on the dispersion properties of vertically heterogeneous media (Arai et al., 2004; Foti, 2002; Tokimatsu, 1995; Strobbia, 2002; Socco and Strobbia, 2004). Every surface wave technique requires firstly an high quality recorded seismograms to be analysed. From the data we can estimate the dispersive properties of surface waves, and then go forward with an inversion process of the dispersion curve, controlling the observed Rayleigh wave dispersion, in order to estimate a realistic shear wave velocity profile. One of the most neglected problem in surface wave analyses is the complexity of the multimodal wave propagation which can lead to modes misidentification (Tuan et al., 2011). Even if several issues must be fronted, e.g. attention must be paid to the presence of subsurface structure more complex than simple one-dimensional layering (Strobbia and Foti, 2006; Vignoli et al., 2009, 2011), the inversion of the dispersion curve remains one of the most critical aspect of all the procedure (see Lai et al., 2005, Maraschini et al., 2010). Commonly dispersion curves are identified as energy density maxima in transformed domains, for example the frequency-wavenumber or the frequency-velocity domain. For each frequency the energy maxima are identified in correspondence of certain modal wavenumbers k or velocities v, and it is commonly assumed that the highest modal k, corresponding to the lowest phase velocity v, belongs to fundamental mode of propagation (Weaver et al., 1982; Karray et al., 2008; Park et al., 1999). Generally common surface wave users consider only the largest spectrum energy frequency by frequency, and the fundamental mode is assumed as dominant even if this is not always verified (Foti et al., 2002; Zhang et al., 2003). Often from surface wave analyses we observe only an apparent dispersion curve which represent the contribution of several modes. The focus of this study is on evaluating the effects of mode mis-identification, in particular as a consequence of the well known ‘osculation’ phenomenon (Malischewsky et al., 2008). Osculation points are point at which two modal dispersion curves get very close and often have the appearance of crossing each other (Cercato, 2009). We want here to show as the osculation phenomenon is strictly related with the polarization of Rayleigh motion in the case of strong impedance contrast in the first subsoil (Boaga et al., 2013). The use of a multi components analyses of motion can be really useful in certain particular geological conditions.

Osculation and velocity contrasts. The errors in terms of depth and velocity of the bedrock can be large if modes identification is wrong, and these parameters are of interest in engineering surveys. When strong impedance contrast is present energy shift from the fundamental to the first higher mode at the so call ‘osculation’ point. In the case of ‘perfect osculation’ even with ideal noise-free modal synthetics, a standard data analysis will extract an apparent continuous curve resulting from the two adjacent modes. But in the applied practice of multi-channels analysis of Rayleigh wave (MASW, Park et al., 1999), given the measurement noise, and the
limited spectral resolution of finite array lengths, this ambiguity can be expected even with larger velocity differences between the two osculating modes.

In order to investigate this situation we test synthetic cases with and without a large velocity contrast. The synthetic datasets (full-waveform seismic records) are generated using the SEM2DPAcK software (Ampuero, 2008). The excitation source is, in all cases, a Ricker wavelet with 20 Hz frequency peak, located at the ground surface and corresponding to a vertical motion. The simulated recorded data are considered reliable up to 50 Hz. At the bottom and at the lateral boundaries we applied Stacey (1988) absorbing conditions. In order to assess purely the effects of the geological conditions which generate mode contamination, we simulated an optimal surface wave acquisition without spatial aliasing and spectral resolution issues, using a 640 m long acquisition array with 1 m receiver spacing.

We consider 3 simple cases of the same subsoil model with a soft layer resting on top of a stiffer half space; we applied an increasing impedance contrast with the aim of generating the well known case of mode mis-identification as in Fig. 1. As in this simple modal synthetic case, a large velocity contrast shows an ‘osculation point’, which is the frequency around the two modes have a ‘kissing’ point. For frequencies smaller than the osculation frequency most energy is located on the first higher mode rather than on the fundamental mode, while for frequencies larger than the osculation frequency, energy remains on the fundamental mode.

The behaviour of the high impedance contrast case can lead to serious misinterpretation of the dispersion curve, with very large overestimation of the bedrock shear wave velocity. Note that this erroneous velocity estimation is acting against precaution, that can have detrimental impacts on seismic amplification assessment. Inverting the apparent curve assuming that the curve belongs to a fundamental mode will lead to a large overestimation of the half-space velocity. This in turn can have a severe impact on seismic response analysis (e.g. Boaga et al., 2011, 2012a).

In practical cases, in presence of coherent and incoherent noise, with the loss of spectral resolution due to the limited array length and the geophone spacing, the identification of the
two separate modes can become even harder. The discontinuity of the picked maxima in correspondence of the osculation point can totally disappear: with a poorer f-k resolution the osculation point is often not recognizable.

The apparent dispersion curve seems continuous in velocity, and the spectral amplitude distribution is continuous as well. The amplitude of the recorded vertical particle velocity results from the smooth transition between the contribution of the fundamental mode and the first higher mode.

**Polarization and osculation.** As discussed by Tuan et al. (2011) and Malischewsky et al. (2008), the osculation phenomenon occurs only for certain impedance contrast cases, and has consequences also for seismic response analysis (Konno and Omachi, 1988; Castellaro and Mulargia, 2009; Mucciarelli et al., 2009; Boaga et al., 2011). High impedance contrasts have, in fact, also a strong impact onto seismic amplification, and are a primary target for the site characterization (Nogoshi et al., 1970; Nakamura, 1989; Fah et al., 2011; Bonnefoy-Claudet et al., 2008; Hobiger et al., 2009).

It is well known that the Rayleigh wave particle motion is elliptical; the ratio between the vertical and the horizontal components, the *vertical ellipticity*, depends on the velocity profile. In a homogeneous half space, the ellipticity is constant for all frequencies, and a function of the Poisson’s ratio. In vertically heterogeneous media, the ellipticity is a function of frequency, and its character is strongly dependent on the velocity contrasts.

The polarization of surface waves is related to the velocity structure, and the signature of the velocity structure on the polarization becomes larger with larger velocity contrast. Therefore several characterization techniques have been devised to take advantage of motion polarization. It can be shown that the magnitude of the ellipticity peak is directly related to the velocity contrast. But we want to show here that also the relationship between polarization and osculation is direct.

In absence of sharp velocity contrasts, modes are well separated and the ellipticity curve is fairly flat. In presence of a strong velocity contrast, the distance between the modes is reduced and the ellipticity peak becomes more pronounced.

It is evident that the cases with large velocity contrast are the most dangerous for the possible mode mis-identification. And in these cases, the ellipticity exhibits behaviour progressively more extreme as the velocity contrast increases. It can be observed as at certain frequency $f_H$ the horizontal particle motion vanishes, and the Rayleigh wave is purely vertically polarized, and one frequency $f_V$ the vertical particle motion vanishes, and the polarization becomes horizontal. These singularities are also points where the direction of the elliptical particle motion changes: from counter-clockwise to clockwise and to counter-clockwise again. The peak of the vertical ellipticity takes place very close to the oscillation frequency. It is important to enhance how the ellipticity discussed here refers to the fundamental mode only. The polarization of the first higher mode has an opposite behaviour, becoming vertical for frequencies smaller than the oscillation frequency.

Large contrasts in the subsurface produce both the mode osculation and a high peak of vertical ellipticity. Therefore, existence of such peak indicates the presence of a large velocity contrast, and the risk of oscillation. The frequencies at which these two phenomena occur are related, even in multilayer models. The oscillation happens because of a drop of amplitude of the vertical component, and below the oscillation the vertical component reaches a zero.

A Monte Carlo simulation of 2000 cases with 3 layers over a bedrock, with a large velocity contrast, is performed and analysed, and the results show the direct relationship between these frequencies (Boaga et al., 2013).

The oscillation happens at a frequency between to the frequency $f^H_H$ of the horizontal polarization and the frequency $f^V_V$ of vertical polarization, the peak of vertical ellipticity curve of the fundamental mode. As a consequence we aim at demonstrating that the mode oscillation
problem can be predicted and solved studying the ellipticity. The use of multichannel, passive single station data and multicomponent active multichannel data can be a practical solution.

**Multi-component surface wave data.**

**Passive data.** Site characterization techniques base on the spectral relationship between horizontal and vertical components of ambient noise have become very popular, being fast and efficient especially in mapping and zonation projects. The Horizontal to Vertical Spectral Ratio (HVSR) has been proven to be an efficient detector of impedance contrasts, even if the nature of the spectral peaks is still debated. Even neglecting the intricacies of the $\lambda/4$ hypothesis (Konno and Omachi, 1988; Lachet and Bard, 1994; Chatelain et al., 2007), body wave contribution and HVSR amplitude interpretation (Bard, 1998), measurements of single station noise is certainly capable of excluding or confirming the presence of a strong impedance contrast (Bonnefoy-Claudet et al., 2008). Consequently, single station noise measurements can also be useful in the identification of a subsoil that can induce modal misidentification in surface wave dispersion study.

It has been proven that, under some hypotheses (or conditions) the average spectral ratio is a reliable measure of the Rayleigh wave ellipticity (Hobiger et al., 2009; Malischewsky and Scherbaum, 2004). With strong velocity contrasts, and with near sources of ambient noise, the noise wavefield is dominated by surface waves, and the spectral ratio can be considered as an experimental ellipticity curve.

We created some simple synthetic ReMi data, using a model with 5 m of sediments ($V_s=300$ m/s) over a bedrock of ($V_s=1100$ m/s). The source distribution is random and uniform in azimuth around a linear array, composed of 48 channels, spaced 2 meters. The resulting frequency-normalized ReMi spectrum is shown in Fig. 2. The generation of pure Rayleigh wave data and the computation of the spectrum follow the approach of Strobbia and Cassiani (2011). We notice that at the osculation point, the different energy ratio between the two modes changes the shape of the averaged spectrum. The peak becomes closer to the higher mode, due to the effect of the energy shift, but the spectrum is more skewed. The inspection of two slices at constant frequency, at 23 Hz and 28 Hz, above and below the osculation frequency, shows the relationship between the spectral maxima and the true wavenumber of the fundamental and higher modes (Figs. 2b and 2c). If some evidence of lower velocity is present, the ambiguities

![Fig. 2 – F-K analysis of synthetic ReMi f-k spectrum generated with 5000 sources uniformly and randomly distributed around a linear 48 channels with 2 m spacing array for (5 m of sediments with $V_s=300$ m/s over a bedrock of $V_s=1100$ m/s). Two slices at 28 Hz and 23 Hz are plotted respectively in panels b) and c). Below the osculation frequency the peak of the spectrum is closer to mode2, thus also these ReMi data suffer from the osculation problem.](image)
of picking such spectrum do not reduce the uncertainty on the extracted dispersion. To pick the fundamental mode, one should peak at 80% of the spectral maximum at 28 Hz, and at 10% of the spectral maximum at 23 Hz. We conclude that also passive ReMi data suffer from the osculation problem. Less ambiguous is the use of a different measurement of the same active wavefield, and the use of the horizontal component can help in solving the osculation issues.

**Active data.** As shown above, the energy shift between modes is related to the different ellipticity for frequencies smaller and larger than the osculation frequency. In particular below the osculation point the Rayleigh wave ellipticity makes the first higher mode much more energetic in the vertical component than the fundamental mode. This implies that if only the vertical component of soil motion is acquired and analysed in the multi channel seismic records, energy seems to shift from the fundamental mode (at frequencies larger than the osculation frequency) to the first higher mode (below the osculation frequency). This can potentially lead to severe errors in dispersion curve identification, but is only due to the traditional use of vertical geophones only. Fig. 3 shows the spectra from the vertical and horizontal component data for the same model of Fig. 2, with a limited length array. The horizontal component allows the identification of the fundamental mode, even below the osculation frequency, with no ambiguity.

Note that the horizontal components do not present the disturbing osculation phenomenon so important in the vertical components. In particular, below the osculation frequency, energy maxima still remain on the fundamental mode, except for the very low frequencies, and thus produce an easily identifiable dispersion curve. This is in agreement with the ellipticity polarization shown in Fig. 8, in which the horizontal component of the fundamental mode maintains high amplitudes below the osculation frequency.

These synthetic results and the supporting theory indicate that the use of horizontal component receivers in multichannel arrays can allow a correct definition of the fundamental mode dispersion down to low frequencies, well below the osculation point. This conclusion can lead to the design of alternative operational practice that can avoid any overestimation of bedrock seismic velocity as a consequence of the osculation problem. The same linear array can be used with multi-component receivers acquiring multiple shots.

**Conclusions.** Below the osculation frequency the f-k spectrum energy maxima of the vertical component of motion do not insist on the only fundamental theoretical mode. This can lead to large errors in the inverted models if modes contribution is not considered, with large over estimation of bedrock velocity. The osculation frequency is directly related to the thickness and the velocities of the layers and a similar behaviour is observed in the theoretical Rayleigh ellipticity. Our synthetic tests show that the osculation frequency is practically the same frequency at which Rayleigh ellipticity
fundamental mode has a minimum. We note how the osculation frequency, at which we observe the shift of energy between fundamental and first higher mode, is the same frequency at which fundamental and first higher mode have similar ellipticity. We described as the fundamental mode below that singular frequency has a predominantly horizontal motion, while first mode has a predominantly vertical motion. If we look only on the vertical component energy seems to shift from the fundamental to the first mode while, on the contrary, the use of horizontal components can help to avoid mode misidentification. This evidence has been experimentally demonstrated in Boaga et al. (2013).

These synthetic results and the supporting theory indicate that the use of horizontal component receivers in multichannel arrays can allow a correct definition of the fundamental mode dispersion down to low frequencies, well below the osculation point. This is the reason to employ a Multi-components Analyses of Surface Wave (McASW). This approach can lead to the design of alternative operational practice that can avoid any overestimation of bedrock seismic velocity as a consequence of the osculation problem. Since one of the main use of shear wave profiles is for the seismic amplification analysis, and that bedrock velocity overestimation could lead to serious misleading, the value of McASW approach here proposed is evident.

Tab. 1 – Properties of the synthetic models.

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Tab. 2 – Subsoil model for the Belluno site.

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References


Park C.B., R. D. Miller and J. Xia, 1999, Multichannel analysis of surface waves. Geophysics; June 1999; v. 64; no. 3; p. 800-808; DOI: 10.1190/1.1444590


Socco L.V., S. Foti and D. Boiero, 2010, Surface-wave analysis for building near-surface velocity models -Established approaches and new perspectives, Geophysics, 75(5); P. 75A83-75A102, 10.1190/1.1374941


