VISCOSOUS DISSIPATION AND TEMPERATURE DEPENDENT SHEAR THINNING RHEOLOGY
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Despite the great progresses achieved in numerical modeling, it is not yet possible to completely model a lava flow. The difficulties lie in the fact that the lava is a fluid with extremely complex rheological behavior, given the dependence, usually non-linear, of viscosity on temperature, on the crystal and bubble content and on the strain rate.

Moreover, the cooling mechanism is the result of different thermal exchanges both external (surface thermal radiation, forced convection, conduction to the base) and internal (axial advection, viscous dissipation, latent heat, internal conduction).

During the flow, the mechanical energy necessary for deformation and flow is dissipated and converted in internal energy, which increases and causes a temperature rise.

Recently, a few attempts have been made to model the viscous dissipation in the heat equation and it has been shown that viscous heating can decrease the flow thickness and increase the flow velocity (Piombo and Dragoni, 2011), it can generate a local increase in temperature with consequent decrease of the fluid viscosity (Costa and Macedonio, 2003) and can trigger and sustain secondary rotational flows (Costa and Macedonio, 2005).

In this study, we numerically solve the dynamic and heat equations with a shear thinning viscosity dependent on temperature, including the viscous dissipation term in the heat equation.

The fluid flows in the $x$ direction in a rectangular channel of width $a=3\text{m}$ thickness $h=1.5\text{m}$ and length $L=50\text{m}$, inclined with slope $\alpha$ and with the cross section parallel to the $yz$ plane. The flow is assumed laminar and subjected to the gravity force. We assume a no-slip at the solid boundary and that pressure changes are negligible with respect to body forces. We consider heat advection in the flow direction $x$ and viscous dissipation. Temperature $T_e$ at the inflow surface of the lava flow is assumed constant and equal to the effusion temperature. The fluid is assumed isotropic, incompressible, with constant density, thermal conductivity, and specific

![Reynolds number contour maps](image)

Fig. 1 – Reynolds number $Re$ at the steady state for effusion temperature $T_e=1000 \, ^\circ\text{C}$ and $\alpha=20^\circ$ for the case study with viscous dissipation. a) Contour maps of $Re$ on the channel surface $z=0$; b) contour maps of $Re$ on the channel bottom surface $z=-h$; c) contour maps of $Re$ on the channel levee surface $y=\pm a/2$. Color gray indicates areas where $Re<Re_c$. d) Vertical profile of $Re$ at the channel outflow boundary ($x=L, y=0$); e) horizontal profile of $Re$ at the channel outflow boundary ($x=L, z=0.5 \, \text{m}$). Dashed lines indicate the value $Re_c=2000$. 

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heat capacity assumed constant so that the thermal diffusivity is constant too. The thermal solid boundaries are modeled imposing a constant heat flux $q_c = 1000 \, \text{W m}^{-2}$. The top surface is treated as a thermal boundary by imposing a cooling for thermal radiation and as a dynamic boundary imposing the free surface condition.

The differential equations are transformed into algebraic equations through the use of the finite volume method (Patankar, 1980; Filippucci et al., 2010, 2013) and the algebraic equations are then solved iteratively. The solution takes into account the coupling between the dynamic and the thermal equations due to both the temperature dependence of the viscosity in the dynamic equation and to the viscous term in the heat equation.

The results indicate that the Reynolds number $Re$ increases in very limited areas of the channel compared to total domain. The areas in which the Reynolds number exceeds the critical threshold $Re_c$ correspond to areas of the domain that heat up, due to the viscous dissipation effect, that lies on the lateral and basal boundaries (Fig. 1). Using this numerical model we could see that the temperature and the velocity growth is not uncontrolled but in a short time interval, which in our case study is about 30 minutes, the temperature reaches a stationary value (Fig. 2).

This result implies that inside the fluid, that flows in the channel, areas in which the flow is in the laminar regime and areas in which the flow is in the turbulent regime can coexist.

The local turbulent state can bring the fluid to develop local vortex. This result is in agreement with other numerical models (Costa and Macedonio, 2003, 2005) who found out that viscous friction causes a local increase in temperature near the walls that, added to the strong coupling between viscosity and temperature, causes viscosity decrease that can lead to the formation of local flow instabilities similar to vortex which cannot be predicted by simple isothermal Newtonian models.

From the observations of basaltic lava flow emplacement, the concept of laminar flow in which all fluid particles move in parallel is certainly not always valid but it cannot even be said that a lava flow is turbulent in the classical sense (Baloga et al., 1995), and the transition from purely laminar flow to turbulence is observed to occur at low values of the

![Fig. 2 – Temperature $T$ as a function of time $t$ in six monitoring points $P$ for $T_e=1000 \, ^\circ\text{C}$ and $\alpha=20^\circ$ for the case study with viscous dissipation (solid line) and without viscous dissipation (dashed line). a) $T$ vs $t$ in the monitoring point $P_1(x=L, y=\pm a/2, z=-0.5 \, \text{m})$; b) $T$ vs $t$ in the monitoring point $P_2(x=L, y=\pm a/2, z=2, z=0)$; c) $T$ vs $t$ in the monitoring point $P_3(x=L, y=0, z=0)$; d) $T$ vs $t$ in the monitoring point $P_4(x=L, y=0, z=-h/2)$; e) $T$ vs $t$ in the monitoring point $P_5(x=L, y=0, z=-h)$; f) $T$ vs $t$ in the monitoring point $P_6(x=L, y=\pm a/2, z=-h)$; g) sketch of the channel cross section with details of the monitoring points.](image-url)
Reynold number. (Keszthelyi and Self, 1998; Costa and Macedonio, 2005). In this work, using a variable Reynolds number, we can conclude that the transition from laminar flow to turbulent flow can take place locally within the channel near the solid boundary, where the temperature gradient rises, in correspondence of which the Reynold number can assume values much higher than the channel isothermal center.

References


