A HYBRID METHOD TO ESTIMATE UNCERTAINTY IN 2D FWI: APPLICATION TO AN INCLUSION MODEL

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Introduction. Full-waveform inversion (FWI) is a valuable tool to derive high-resolution models of the subsurface having available a reliable macro-model containing the correct large-wavelengths of the model. FWI is generally cast in the framework of deterministic approaches, and hence it returns a single best-fitting model in which no information is given regarding the associated uncertainties of the model parameters. However, in practice, many inverse problems are ill-posed meaning that many solutions explain observations and theory equally well. Therefore, estimating the uncertainties that affects the final result of an inverse problem provides valuable insights on the equivalence region of the solutions (Fernández et al., 2012).

Sajeva et al. (2016) proposed a workflow to determine uncertainties in two-dimensional FWI. This workflow can be divided in two parts. In the first part, a genetic algorithm combined with a Gibbs Sampler (GS) (Sambridge, 1999; Aleardi and Mazzotti, 2016) derives a low-resolution P-wave velocity (Vp) model and its uncertainties. In the second part, the PPD derived by the GS is used to perform a set of full-waveform inversions using iterative descent-based techniques, which in turn are used to perform a statistical analysis of the final high-resolution solution. In this work, we apply this method to a simple example model modified from Mora (1989), that consists in a background gradient model with a horizontal reflector and a spherical inclusion.

Theory. Genetic algorithms (GAs) are a class of randomized search methods that can be applied to large-scale optimization problems. They treat models collectively, and they make evolve the ensemble of models (or population) toward new generations with lower misfit by means of selection, recombination, and mutation. Since their introduction (Holland, 1975), several implementations of GAs, both binary and real coded, have been proposed. A recent and promising version of GAs is the Breeder Genetic Algorithm (BGA), an efficient real-coded algorithm in which the optimisation is mainly guided by the selection and recombination steps (Schlierkamp-Voosen and Mühlenbein, 1993).

GAs are not a Markov Chain Monte Carlo (MCMC) method, consequently, the ensemble of models explored during a GA inversion is not sampled according to the posterior probability distribution (PPD). Therefore, a biased estimation of the PPD is produced if it is directly computed from the collected models and their associated likelihoods. From a Bayesian point of view the posterior probability distribution is the solution to the inverse problem, and it contains all information available on the model. The PPD calculation depends on the data, any prior information, and the noise statistics (which is assumed known). At any point \( m \) in model space \( M \) the PPD is given by:

\[
P(m|d^0) = \frac{k}{} \rho(m)L(d_o|m),
\]

where \( \rho(m) \) is the prior probability distribution, \( L \) is the likelihood function, \( k \) is a normalizing constant, \( d_o \) is the observed data set. To convert the ensemble of GA models to a non-biased PPD, we make use of the procedure of Sambridge (1999), that resamples the model space using the Gibbs Sampler. In more details, the model space is divided into Voronoi cells, each one associated with a single GA model and its likelihood. This constructs a multi-dimensional interpolant that is resampled using the Gibbs sampler.

Method. The inversion procedure that we use is composed by the following steps:

1. perform a global inversion using the BGA, and collect all the explored models and their misfit;
2. appraise the entire ensemble of BGA models using the Gibbs Sampler following the procedure of Sambridge (1999). This step returns the uncertainty affecting the low-resolution velocity model derived by GA;
3. use a MCMC algorithm to extract a sufficiently large set of models from the PPD estimated at the previous step;
4. apply local FWI to each model obtained at step 3;
5. apply a non-parametric method (e.g., the kernel density estimation) to the entire set of final models derived at the previous step. This step gives the final PPD that represents the uncertainties associated with the final Vp models.

Note that the second step yields a non-biased PPD which expresses the uncertainties affecting the best-fitting BGA model. This method returns the 1D marginal PPD for each model parameter. To mitigate the so-called curse of dimensionality in the GA optimization, we reduced the number of model parameters by resampling the prior model onto an irregular grid with cell sizes chosen according to seismic resolution criteria, that is, proportional to a quarter of the dominant wavelength for the vertical resolution and proportional to the first Fresnel zone for the horizontal resolution. See Sajeva et al. (2016) for more details.

**Synthetic example.** We apply the method to an acoustic inclusion model similar to the one introduced by Mora (1989). This model is constituted by a spherical homogenous inclusion in a background velocity model characterized by a constant gradient with depth and a deep reflection (see Fig. 1a). For the forward modelling we use the finite-difference method, with accuracy of second order in time and fourth order in space, a vertical and horizontal space step of 48 m, a time step of 4 ms, and a 6 Hz Ricker wavelet as the source signature. The acquisition geometry consists of 31 sources equally spaced at the surface, which illuminate all the evenly spaced 127 receivers at the surface. To evaluate the misfit, we use the L2 norm applied to low-pass filtered (0-3 Hz) and trace-by-trace normalized data.

As prior information, we use a simple 1D Vp model with velocity linearly increasing with depth from 1500 to 3000 m/s. This model is used to centre the GA inversion ranges and also to build the irregular GA. The grid and the 1D model are shown in Fig. 1b. This grid has 26 nodes. These nodes are bilinearly interpolated to the finite-difference grid for the forward-modelling.

In the inversion, we performed 16k model evaluations and the final best-fitting model is shown in Fig. 2a. This result may be considered a good macro model, since it contains the long-wavelengths of the true model. In addition, we desire to appraise the entire ensemble of GA models to quantify the uncertainty affecting the BGA solution of Fig. 1c. To this end, we employ the GS. Fig. 1d shows some of the resulting 1D marginal PPDs (first row: from left to right; second row, central line from top to bottom).

![Fig. 1](image_url)

Fig. 1 – a) The true inclusion model; b) the “starting model” superimposed with the grid nodes; c) the final model after GA; d) the uncertainties associated with some model parameters (first row: at surface from left to right; second row, central line from top to bottom).
right, second row: from top to bottom). Note that, moving from the center to the lateral edges of the model, the PPDs become broader and multimodal. Analogously, moving from the top to the bottom of the model, the PPDs suffer multimodality and widening. These characteristics are in agreement with the expected loss of information due to the poorer illumination in the lateral and deep parts of the model.

Next, we extract 200 models from the PPD derived by the BGA inversion and the Gibbs sampling and we employ them as starting models for LFWI. We use the time-domain steepest-descent method, with 5 iterations at 4, 5, 6, 8, and 10 Hz. The mean value of all the resulting final models is displayed in Fig. 2a and it can be directly compared with the true model of Fig. 1a, finding a satisfactory match. Fig. 2b shows the approximate 99% confidence interval of the set of final models. Note that the highest uncertainties are mainly localized where seismic illumination is poorer. Fig. 2c shows the 1D marginal PPDs, at the same positions indicated in Fig. 1d. Again, note the loss of resolution from the centre to the lateral edges, and from the shallow to the deeper parts of the model. Comparing Fig. 1d with Fig. 2c, we observe that the multimodal behaviour disappears and that the distributions are narrower. We also show three velocity profiles (Fig. 3a) from the far left to the centre of the model (at offsets 0 m, 2000 m, and 4500 m); the set of starting models is displayed by the grey beam, the set of final models is displayed by the cyan beam, and the true model by the black line. The velocity profiles (Fig. 3) and the comparison between the distributions (Figs. 1d and 2c) highlight several points: 1) the set of models (grey beam) resulting from GA FWI reconstructs well the low frequency trend of the true velocity model; 2) the loss of resolution with depth and near the edges of both the GA and LFWI solutions, 3) the narrowing of the distributions after local FWI, and (4) the improvement in the estimation of the true model after local FWI, especially for the central part of the model where the seismic illumination is higher. Finally, note that the marginal PPDs (Fig. 2c) do not display multimodal shapes, thus demonstrating that the whole set of starting models converge toward the same model-space region, and that, where the model is sufficiently illuminated by the wave-fronts, the true Vp values lay within the final PPDs.

Conclusions. Standard approaches for determining a starting model for FWI, such as reflection tomography, provide a single velocity macro-model. The method that we investigate
here, which involves genetic algorithms (GAs) and a Markov Chain Monte Carlo resampler, provides also the uncertainties associated with the maximum a posteriori model, that is, it returns Posterior Probability Distributions (PPD) of the model parameters. These PPDs in turn can be propagated via Local FWI (LFWI) to obtain high-resolution estimates of the PPDs. To make the GAs inversion computationally feasible, we apply a two-grid approach that uses a coarse grid with a variable grid spacing in the optimization phase and a fine regular grid in the forward modelling phase. To obtain unbiased estimates of the GA PPD we importance sample the GA-model ensemble by means of a Gibbs sampler (GS). We validate the reliability of the GA FWI+GS approach by checking the uncertainty propagation from the starting models to the final LFWI models. This procedure has been tested on a variation of the 2D acoustic inclusion model of Mora (1989). It results that the multimodal and wide marginal PPDs derived from the global optimization (GA FWI+GS) become unimodal and narrower after local FWI and, in the most illuminated part of the subsurface, contain the true model parameters. This indicates that the set of models derived from the GA+GS PPD produces an ensemble of starting models that enable local FWI to converge toward the same model-space region. The practical use of this method is presently limited to 2D and acoustic inversions due to the intensive computational cost of this procedure.

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References


