2D APPROACH FOR SOLVING SELF-POTENTIAL PROBLEMS: MODELING AND NUMERICAL SIMULATIONS
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Introduction. The self-potential (SP) method is a well-established geophysical technique that has been applied, since its inception in the early 19th century, to mineral exploration, oil well logging, geothermal exploration and more recently hydrogeologic, environmental and engineering investigations. In the past, this geophysical technique has been considered as a qualitative method, but nowadays, different quantitative interpretations have been proposed to define the geometry and density of the causative source. Sill (1983) was the first to introduce a physics-based approach to simulate numerically the self-potential using the finite difference method. Later, self-potential signals have been modeled by implementing the finite element approach (Soueid Ahmed et al., 2013) and the finite volume method (Sheffer and Oldenburg, 2007). The two main contributions to the SP signals are the streaming potential associated with the drag of the excess of charge by the flow of the pore water and the “electro-redox” effect associated with redox potential gradients. Most recent applications of the SP method include investigations aimed at reconstructing the hydraulic head variations caused by pumping tests (Titov et al., 2015) and detecting leakage paths in earth dams. The SP data have been also used to locate subsurface cavities (Jardani et al., 2006) and the preferential flow pathways in geothermal field and active volcanoes. Several other recent efforts have focused on applications in mapping and assessment of contaminant plumes (Rittgers et al., 2013). In this work, we introduce a two dimensional numerical modelling tool in MATLAB for predicting the SP response. Self-potential signals are obtained by starting with the solution of the groundwater flow or the electro-redox problem, then computing the source current density, and finally calculating the electrical potential. Selected case studies are presented in order to simulate both the electric field resulting from the existence of a leak in the dam and SP signals associated with a pumping test in an unconfined aquifer. In addition, to illustrate the efficacy of the algorithm, field data are examined.

Theoretical background. A MATLAB code based on the finite element method is implemented to provide numerical 2D modelling of SP. The approach described here is based on the well-known constitutive equation:

\[ \nabla \cdot (\sigma \nabla \psi) = J \]

where \( \sigma \) (in Sm\(^{-1}\)) and \( \psi \) (in V) are the electrical conductivity and the electrical potential,
respectively. The term (in Am$^{-3}$) represents the volumetric source current density.

The proposed Matlab program employs the FEM to calculate a numerical solution which approximates the exact solution to the two-dimensional Poisson problem:

$$\int_{\partial\Omega} W \sigma \nabla \psi \cdot dS - \int_{\Omega} \sigma \nabla \psi \cdot \nabla W d\Omega - \int_{\Omega} W J d\Omega = 0$$

where $W$ is a “weighting function” and $\Omega$ represents the domain in which the condition is enforced.

The first term is related to the boundary conditions of the problem while the second and third ones indicate, respectively, the stiffness matrix and the source term.

For the implementation, the problem is discretised using the standard Galerkin method; the integrals on $\Omega$ for the discretized domain become:

$$\sum_{n=1}^{N} \int_{\Omega_n} [\sigma \nabla \psi \cdot \nabla \Phi_n - J \Phi_n] d\Omega = 0$$

where $n$ represents a generic element and $N$ is the number of elements in the solution domain.

**Cases study analysis.** The algorithm is validated by case studies presented by other authors. We show a comparison of the results obtained from the same dataset with the developed numerical code and other software programs. The first case study discussed concerns a pumping test in an unconfined aquifer. The model of the test site and details of the experiment modelled were described by Titov et al. (2005). For the hydraulic modelling, MAXSym, a MATLAB tool which is designed specifically to simulate axisymmetric flow (Louwyck et al., 2012), was utilized. Taken into account that we used the parameters determined on the basis of the Theis solution (Theis, 1935), we considered the aquifer confined in the course of the pumping test (with the initial head at 48 m above the datum). Supposing the aquifer was homogeneous and of infinite lateral extent with fully penetrating well, drawdown $s$ was calculated analytically using the solution given by Butler (1988).

Then, we modeled SP signals on the basis of the material properties reported in Titov et al. (2005) (Fig. 1). We used the no-flow condition on the ground surface, and the condition of zero electrical potential on the other sides. The right boundary of the model was located far away from the pumping well in order to ensure that the boundary conditions have no influence on the computation of the SP. The potentials were calculated relative to infinite point. We obtained a radial SP distribution in the vicinity of the pumping well at the end of the pumping phase in accordance with the results reported from the authors. As is shown in Fig. 1, the positive electrical source is centred on the pumping well, while negative sources are located far away under transient regime of the groundwater flow.

Furthermore, to test our
algorithm, we performed a numerical simulation for a synthetic embankment dam with a simple 2D geometry (Fig. 2). A leakage is reproduced by adding a permeable pipe located inside the dam core that simulates the existence of a preferential ground water flow pathway with permeability much higher than the permeability of the surrounding area. The petrophysical properties used in the simulation are reported in Ikard et al. (2012). The self-potential synthetic data were referenced to a point located at infinity. Boundary conditions were defined as follows: water head pressures were imposed along the lateral boundaries using a Dirichlet condition. At the bottom of the dam and at the ground surface we imposed an insulating boundary condition (n·j=0) where n is the unit vector normal to the ground surface).

![Fig. 2 – Triangular meshes used to discretize the domain for the finite element simulation.](image)

For the electrical problem, the Neumann boundary condition was imposed at the insulating air-ground interface and a Dirichlet boundary condition was imposed at the other boundaries. The numerical model was performed for steady-state flow conditions with a leakage. The groundwater flow due to the hydraulic gradient between the upstream and downstream of the dam is illustrated in Fig. 3. The hydraulic gradient and the average velocity in the conduit resulted, respectively, on the order of 0.17 and on the order of 0.017 ms\(^{-1}\). The magnitude of the simulated self-potential signals is in the same range than those obtained by the authors using a commercial finite element code, COMSOL Multiphysics.

Using dataset from other studies as the reference, we have demonstrated that the developed numerical code implemented in MATLAB performing the forward modeling for self-potential works well and its numerical solutions are reliable. In fact, the result of the sign and the magnitude of the computed self-potential signals agrees well with the values of field and synthetic data. Future works will include development of a strategy to solve the inverse problem in order to reconstruct 2D distribution of the source current density responsible of the observed self-potential anomalies.

![Fig. 3 – Solution of the Darcy velocity obtained by solving the groundwater flow problem.](image)
References


Theis C. V., 1935: The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. American Geophysical Union Transactions, 16, 519–524.
