This contribution describes GPR experiments related to two cylindrical columns built close to the Institute for Archaeological and Monumental Heritage IBAM-CNR, office of Lecce, Italy. The investigation of cylindrical structures is of interest for the non-invasive diagnostics of monumental buildings (Leucci et al., 2007), modern circular pillars made-up of reinforced concrete, or even tree logs (Butnor et al., 2009). In particular, a GPR investigation is able to provide information about the possible presence of internal voids, fractures, metallic hinges or reinforcement rebars.

When GPR data are collected along the height of the column, it is often acceptable to process the data as if they were gathered on a half-space, as done in (Leucci et al., 2007). Anyway, it is also of interest to gather data along circular profiles at a fixed height (Santos-Assuncao et al., 2014), that can provide clear images of the cross-section of the column or of the pillar. In this case, the ray of curvature of the investigated surface is usually of the order of the probing wavelength. As a result, it is not possible to exploit the model based on a flat surface, which is at the basis of most commercial codes for GPR data processing (e.g. Reflexw and GPRslices).
An additional problem is related to the representation of the data, which are provided by the GPR along a rectangular abscissa-depth time grid (B-scan). This is not much troublesome and can be solved by a suitable wrapping of the solution. In this study, after zero-timing the data, we take one half of the recorded time depth up to the reflection from the opposite point of the circumference of the column with respect to the position of the antenna. In this way, we approximately retain the data up to the return trip time from the antenna to the centre of the circular cross section of the column (Masini et al., 2010).

A more substantial problem about the data processing remains, and in particular about the development a focusing algorithm suitable for the imaging task. In this respect, migration algorithms usually exploited in GPR data processing are implicitly based on the hypothesis of a flat surface, and on the hypothesis of a straight observation line along it (Persico, 2014). Here, we adopt a linear inverse scattering algorithm based on the Born approximation and accounting for the geometry of the scenario as well as for the measurement configuration. A 2D geometry is assumed and the pertinent incident fields and Green’s function are considered for the case of a homogeneous scenario. In particular, the incident field and the Green’s function are expressed by the Hankel function of the second kind and of zero order. The imaging problem is recast as the inversion of the following integral relationship:

\[
E_s(\vec{r}) = \iint_{\mathcal{C}} \left( k |\vec{r} - \vec{r}'| \right)^2 \chi(\vec{r}') d\vec{r}'
\]

where \( E_s \) is the scattered field at the measurement position \( \vec{r}^* \) (source-observation point) and \( \chi \) is the contrast function, which is the unknown of the problem expressed as the relative difference between the permittivity of the targets and that of the surrounding medium. \( k \) is the wavenumber within the column. In Eq. (1), several unessential factors outside the integrand are neglected.

The estimation of the reference permittivity [and consequently of the wavenumber to be put into Eq. (1)], is provided by the flat reflection attributable to the opposite side of the columns, which occurs at a known depth. Of course, it would be improper to make use of diffraction hyperbolas, because the curvature of the columns surface distort them. The scattered field data are found in an approximate way from the GPR data after zero timing and a background removal plus a cut time up to the alleged centre of the columns as above said. Then, the data are transformed in frequency domain and inverted by means of an inversion approach working under a multi-monostatic/multi-frequency configuration.

The data have been gathered with a Ris Hi mode system manufactured by IDS equipped
with 2 GHz antennas. A positioning system has been set-up so to follow the observation line in a precise way with regard to the height of the antennas from the ground. The photo on the left of Fig. 1 shows the test site comprising two columns while the drawing on the right depicts internal scheme of the anomalies displaced within the column considered for the experiments.

Fig. 2 illustrates the scattered field radargram related to a circular GPR profile gathered at a height of 136 cm above the ground. A clear diffraction hyperbola originated by the void cylinder (see right picture in Fig. 1) appears in the image. The tomographic reconstruction corresponding to the dataset in Fig. 2 is reported in Fig. 3. This last confirms the effectiveness of the considered data processing algorithm in detecting the presence of defects in the structure.

In conclusion, an ad-hoc test site has been built and an algorithm for the data processing aiming to investigate circular structures has been implemented and some first experimental results have been reported. The work is in progress and future developments will regard the possibility of gathering and processing data with more antennas, as well as the development of a more practical positioning equipment.

Acknowledgements The GPR measurement system was implemented within the research project PROdotti, MEtodologie e TEcnologie Originali e Sostenibili per la diagnosi, la conservazione e la comunicazione dei beni culturali PROMETEOS, funded by the Puglia Region.

References
