Background. The resolution of satellite-derived global gravity models (GGMs) is adequate to resolve the mass distribution in the continental crust, the strong density contrast at the crust-mantle boundary (CMB), and the undulations of the lithosphere-asthenosphere boundary (LAB). These aspects suggest that GGMs can be promising tools in modelling the deep thermal state of the lithosphere, the heat transfer regimes involved and the heat flow through the Earth surface. The directly measurable near-surface temperature field is largely influenced by ongoing geo-dynamics and near-surface processes, both of which have shorter characteristic timescales than the one needed by purely conductive thermal diffusion to reach steady-state equilibrium in the lithosphere. Heat flow measurements are also costly, their distribution is often biased towards areas of increased interest (e.g. those with high fluxes, exploited for high-enthalpy geothermal energy) and public access to data is an issue. Collecting and harmonising the published datasets to a common standard is an effort spanning multiple decades (Lee and Uyeda, 1965).

Gravity and geoid anomalies have already been integrated in multi-observable modelling strategies, and show a satisfactory resolving power for investigating the nature of lithospheric inhomogeneities (Fullea et al., 2009). Still, gravity data alone—which has an unmatched global sampling regularity—can already provide estimates independently from other geophysical data, before integration. A relationship between the lithospheric mass distribution (inverted from density contrasts) and models of its thermal state must rely on laws connecting density and thermal parameters (i.e. radioactive heat production, thermal conductivity, boundary conditions), and a set of hypotheses on the heat transport mechanisms involved. A key factor is the radioactive heat production (RHP) occurring in the crystalline continental crust, which exhibits a 50-fold increase against sub-continental mantle content in U, Th, K (Vilà et al., 2010).
and is a major component of the surface heat flow even when superimposed with concurring near-surface disturbances (Fremark et al., 2015).

Estimating the distribution of these elements occurring throughout the continental crust is not a trivial task, since direct and indirect observations (outcrops, xenoliths, and tomographies, e.g. Rudnick et al., 1998; Jaupart and Mareschal, 2011; Huang et al., 2013) indicate that any simple relationship between crustal thickness and heat production (e.g. Lachenbruch, 1970) is complicated by the large intra-crustal compositional variability. For such reasons stochastic approaches are commonly employed, either exact solutions (Srivastava and Singh, 1998) or random modelling (Jokinen and Kukkonen, 1999), and the results are commonly described with their probability density function.

Apart from parametric uncertainty, the entity and predictability of the relationship between crustal thickness and total heat production is difficult to evaluate on itself, due to aforementioned superposition of effects in the observed surface heat flow.

**Method.** Applying a scaling relationship between the undulation of a gravimetric CMB, inverted from a global gravity model, and bulk heat production is a straightforward operation that can already provide a useful estimate, albeit characterised by large uncertainties: up to 30 mW/m² of interquartile range for a 45 km thick crust (Pastorutti and Braitenberg, 2017). From this approach, we can get insights on the entity of the crustal component of the surface heat flow. It is useful in partitioning different thermal regimes, thus helping the interpolation of existing heat flow measures and their downward continuation, attenuating the effect of incorrect extension of local contributions at large distances (i.e. separating the component due to upper crustal emplacements from the signal due to the variation in heat flow from the mantle).

We analyse the heat flow prediction with a set of synthetic tests, for which we developed a versatile framework for joint gravity and temperature modelling.

The thermal forward modelling part is based on a 3D finite-difference forward modelling solver, on rectangular domains, with non-homogeneous heat production and conductivity, which was written for this purpose. It solves the steady state diffusion equation in the form, where \( k \) is the thermal conductivity, \( A \) is the heat production per unit of volume, and \( x \) is the position vector. The temperature and pressure dependence of thermal conductivity is taken into account, iteratively, using the simple relationships of Chapman (1986) and Schatz and Simmons (1972) for the crystalline crust and lithospheric mantle, respectively. The gravity forward modelling is done with a prism based algorithm, while the inverse modelling relies on an iterative constrained inversion routine (Braitenberg et al., 2007).

The domain-box is designed to represent a portion of the lithosphere, under a flat-Earth approximation, with a flat top and bottom boundary. The top is fixed at \( T(0) \), the surface temperature; the bottom is a flat surface in the upper asthenosphere, it can be alternatively set as a temperature or heat flow boundary condition, which can be iteratively varied to obtain the required LAB. The sediment thickness from the top boundary to the crystalline basement is considered to be known and its gravimetric effect is forward modelled and stripped.

**Results.** We devised the above configuration—which allows the fast prototyping of models with any parameter distribution—to evaluate the joint temperature-gravity effect of different layered geometries and of disturbing bodies in a reference lithosphere; to quantify the required instrumental sensitivity (i.e. the detectability in the measured gravity gradient at orbital altitude); and to test the suitability and effect of the fundamental assumptions. The issue we enquired here regards this last aspect: we assume a relationship between a gravimetric crustal thickness and crustal heat production—to what extent is this adequate to predict the surface heat flow? We revisit the traditional linear model (Jaupart, 1983; Nielsen, 1987) by including the uncertainty due to inverting the effect of the crustal inhomogeneities to a single CMB.

Our analysis shows how concurring effects result in complex phenomena, even in these simplified synthetic conditions. Given a certain a constant lithospheric thickness, an increase in the crustal (radiogenic) heat flow component (due to a thicker or more enriched crust)
results in a decrease in the sub-crustal heat flow, while a constant sub-crustal heat flow requires thermally thinner (i.e. warmer) lithosphere. This process is observed even when the temperature dependence of thermal conductivity -an inverse relationship- is not accounted for. We can also observe the distortion of the surface heat flow footprints produced by heat sources at different depths caused by lateral and vertical inhomogeneities in thermal conductivity and the thermal refraction phenomena involved.

The known limitations of the linear model, even in an ideal case, are shown in Fig. 1, which is an example output for a 2D section. Line a and b are the true relationships for a constant 40 and 50 km crust, respectively: by imposing a constant lithospheric thickness, (the intercept)
Tab. 1 - Fit parameters of the linear relationships of Fig. 1.

<table>
<thead>
<tr>
<th>ID</th>
<th>description</th>
<th>slope [10^6 W/m²]</th>
<th>intercept [10^3 W/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>40 km crust, true relationship</td>
<td>1.13</td>
<td>15.82</td>
</tr>
<tr>
<td>b</td>
<td>50 km crust, true relationship</td>
<td>1.13</td>
<td>12.29</td>
</tr>
<tr>
<td>c</td>
<td>using $Q(SFC)$ for 40 and 50 km crust, apparent rel.</td>
<td>0.79</td>
<td>29.39</td>
</tr>
<tr>
<td>d</td>
<td>using $Q(CMB)$ in the transition zone, apparent rel.</td>
<td>0.59</td>
<td>38.42</td>
</tr>
</tbody>
</table>

decreases for a thicker crust, while the slope is the same, which is consistent with their identical composition (for values see Tab. 1).

The apparent relationships we obtain by fitting the surface heat flow values at the two markers (undisturbed by the transition, line c) or along the transition zone (line d) underestimate the crustal production and overestimate .

We also test the effect of inversion to an apparent gravimetric CMB of the gravity anomaly of a sill-like disturbing body (80x2 km), more radioactive (+2 $\mu$W/m²) and buoyant in respect to the reference crust (-200 kg/m³). Using the apparent CMB for a thermal forward modelling skews the surface heat flow prediction up to 4 mW/m² residual: a small quantity compared with the uncertainties involved, but enough to significantly alter the fit of a linear relationship (errors of up to -0.76 $\mu$W/m³ in average heat production and +30 mW/m² in ).

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References


